AN ANALYTICAL STUDY OF SEPARATED FLOW ABOUT A CIRCULAR CYLINDER

by

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THESIS

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ABSTRACT

The problem of separated flow about a circular cylinder is studied analytically through the use of two potential flow models. Following a detailed review of the present state of knowledge, mathematical formulation of the models and appropriate computer programs are presented. These models are shown to be capable of simulating the asymmetric vortex shedding.

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TABLES OF SYMBOLS AND ABBREVIATIONS

ENGLISH

C_D	Drag coefficient; defined by equation (1)
C_P	Pressure coefficient; defined by equation (27)
c	Radius of circular cylinder or sphere
D	Differential operator
$\frac{dq_i}{dt}$	Cartesian tensor notation for Eulerian total derivative
F_D	Drag force on circular cylinder
F_L	Lift force on circular cylinder
f_i	Cartesian tensor notation for body forces
g	Acceleration of gravity
i	$\sqrt{-1}$
K	A free parameter in Free Streamline Theory
M	Mach number or number of free vortices
N	Total number of free and attached vortices in a flow field
P	Pressure
q_i	Cartesian tensor notation for the three axial velocities; $q_1 = u$; $q_2 = v$, and $q_3 = w$
Re	Reynolds number; for circular cylinder, $Re = \frac{2 U_\infty c}{\nu}$
S	Strouhal number; for circular cylinder, $S = \frac{2c}{U_\infty T}$
T	Strouhal period; U_∞ T = length of wake generated in one period
t	Time
U_∞	Velocity of the ambient flow; could be time dependent
u	Instantaneous velocity in x-direction of a two or three dimensional cartesian system

V	Instantaneous total velocity vector
v	Instantaneous velocity in y-direction of a two or three dimensional cartesian system
W(z)	A function of complex variables; defined by, $W(z) = \psi(x,y) + \phi(x,y)$
w	Instantaneous velocity in z-direction of a three dimensional cartesian system
Z	Complex variable; $z = x + iy$, an undefined location; for example, $z_n = x_n + iy_n$ could be the location of the nth vortex

GREEK

Γ	Strength or circulation of a vortex; defined as the line integral of the tangential velocity component around a closed curve, or defined as the integral of vorticity over a surface bounded by a closed curve
γ	Specific weight; defined by, $\gamma = \rho g$
ϵ_{ijk}	Cartesian tensor permutation symbol
ζ_i	Cartesian tensor vorticity vector; defined by equation (20) in Section II
θ	An angle measured from the positive x-axis
μ	Absolute viscosity
ν	Kinematic viscosity; defined by, $\nu = \frac{\mu}{\rho}$
ρ	Density
ϕ	Velocity potential function
ψ	Stream function

SUBSCRIPTS

i	Cartesian tensor index; i = 1,2,3; or indicates quantity pertains to an image vortex
j	Cartesian tensor index; j = 1,2,3
k	Cartesian tensor index; k = 1,2,3
m	Vortex counter index; m = 1,2,.....,M
n	Vortex counter index; n = 1,2,.....,N

ACKNOWLEDGEMENTS

To Professor T. Sarpkaya, I owe the greatest help and inspiration in theoretical and day to day guidance. Were it not for his original ideas, this thesis would not have been undertaken. To Professor G. Cantin, I express my special thanks for his help in numerical techniques.

I. INTRODUCTION

The postulation of an analytic solution for a viscous fluid flow about a circular cylinder has perplexed hydrodynamic investigators for several centuries. Although notes left by Leonardo da Vinci (1452-1519) indicate more than a qualitative interest, it was not until the early seventeenth century, during the evolution of Potential Theory, that the first formal attempts were made at quantitative analysis. Though there was some understanding of viscous effects during this period, they were basically qualitative and were not taken into consideration when in 1750 D'Alembert found that:

> The force on a body in steady motion relative to an otherwise undisturbed ideal fluid is zero. [1][1]

This result, classically known as D'Alembert's Paradox, was of course unsatisfactory. Decades have elapsed since investigators found the qualitative reasons for the erroneous results given in this and other cases by Potential Theory.

Philosophically speaking, to the engineer the utlimate goal of any analytic procedure is to provide a power of prediction for design applications. As yet this capability is generally only realizable from experimental investigations and then only for certain flow characteristics. For a circular cylinder immersed in a steady cross-flow as shown in Figure 1, the drag force can be calculated from:

$$F_D = \rho c U_\infty^2 C_D \qquad (1)$$

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[1]The bracketed numbers refer to the item number in the List of References.

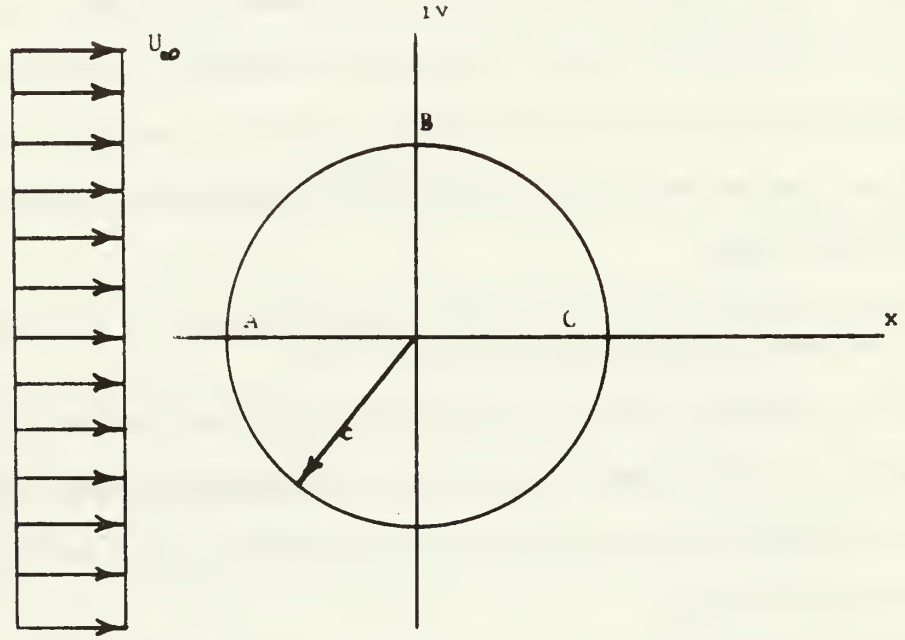

Figure 1. Circular Cylinder Immersed in a Steady Fluid Cross-Flow and Indicating Relationship to a Complex Coordinate System.

C_D is determined experimentally as a function of the Reynolds number as shown in Figure 2. Except for $Re < 1$, there is presently no general analytical method available to generate these results. This exception will be discussed in more detail in Section II. The primary point involved here is that for each geometric shape, or group of geometric shapes immersed in a steady flow field, there would have to be an experimentally found C_D curve for each case. In fact, for an incompressible fluid, the form of equation (1) would have to be more general:

$$F_D = f(Re), \tag{2}$$

and for a compressible fluid:

$$F_D = f(Re, M). \tag{3}$$

For all but the simplest geometries, the definition of Re and M are in themselves difficult problems; not as to their general meanings, but as to specific combination of the physical parameters for a given application.

Though used only as an illustrative example above, the analytical determination of C_D has more subtle implications than those previously indicated. It requires a complete understanding of separated flow, particularly from the aspect of vorticity generation in the boundary layers before separation. It is this point that presents the greatest problem in the postulation of a mathematical model; for it is assumed that if a general analytical solution can be found for C_D in time-dependent flows, then analytic determination of all or most of the other flow characteristics will follow. Also, since a circular cylinder and its surrounding flow field can be easily represented in two dimensions

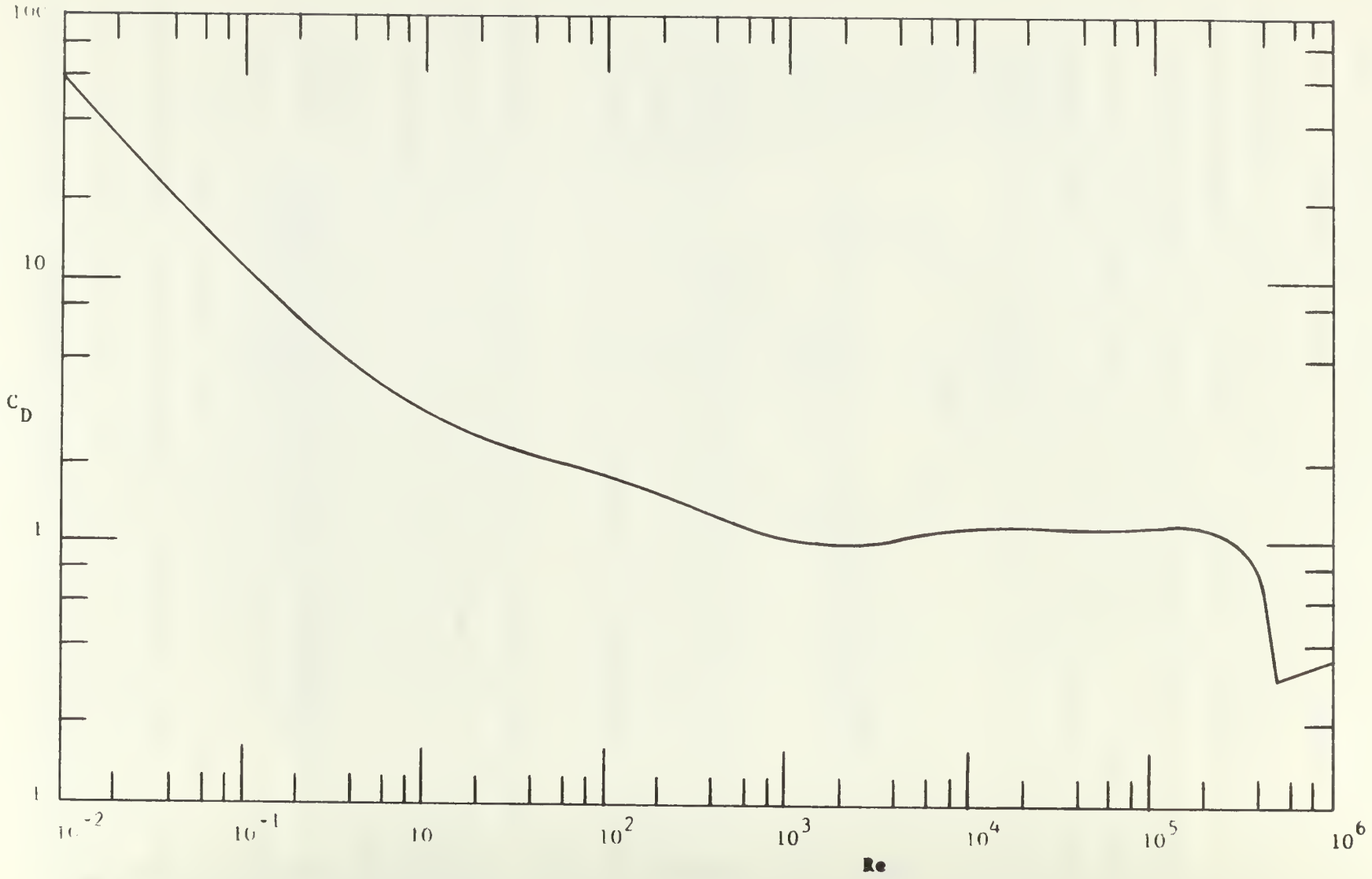

Figure 2. Log-log Plot of Drag Coefficient Versus Reynolds Number for a Circular Cylinder Immersed in a Steady Cross-Flow. [2]

by a function of complex variables, it is possible to easily transform the cylinder into other two-dimensional shapes using the technique of conformal mapping. For inviscid, incompressible fluids, this technique has been used to great advantage particularly in aerodynamics, however external knowledge of boundary layer effects was needed.

So far, the discussion has centered around total drag. For future analysis it is necessary to break this into its two classical components, skin friction and form drag. The skin friction is the integral of the shearing stresses over the surface of the body and is dependent on the action of the fluid in the boundary layer. The form drag is equal to the integral of the forces normal to the surface of the body and is assumed to be dependent only on the inviscid fluid action outside the boundary layer. This assumption is made since an order of magnitude analysis of the Navier-Stokes equations indicates that the pressure gradient through the boundary layer is negligible as compared to other effects. The practice of considering the fluid outside the boundary layer as inviscid is due again to an order of magnitude analysis of the Navier-Stokes equations. This time, when considering only the region outside the boundary layer, the equations reduce to the inviscid equations of motion and permit the application of Potential Theory to this region. Note that large Reynolds numbers have been assumed in both instances.

Following the above reasoning further, leads to the conclusion that if no separation of the boundary layer occurs, the form drag is essentially zero since there is no disturbance to the external inviscid flow. At very low Reynolds numbers, this conclusion is reasonable and can be considered a basis for the famous "creeping motion" solution of G. G. Stokes in 1851. In this case skin friction predominates. At larger

Reynolds numbers, however, there is boundary layer separation and form drag becomes important. The external flow field is disturbed by regions of intense vorticity which causes a decrease in pressure behind the body as compared to the undisturbed inviscid solution. It is evident then, that without considering the direct shear force effects of viscosity as represented by skin friction, and the indirect effects represented by vorticity regions, the qualitative concepts reduce such that D'Alembert's quantitative solution results.

Earlier in this discussion, mention was made concerning the practice of considering the undisturbed fluid outside the boundary layer as inviscid. This requires amplification and a word of caution. In 1908, H. Blasius presented a solution to the boundary layer on a circular cylinder. As far as his paper was concerned the method of solution, rather than the result, was the most important contribution. Briefly, it involves the use of a series expansion for the free stream velocity just external to the boundary layer.[2] If this velocity is taken as that found for incompressible, inviscid flows derived from Potential Theory on the surface of the cylinder, the angular distance between the upstream stagnation point, and the boundary layer separation point is found to be 108.8 degrees; the center of the cylinder is taken as the origin.[3] If experimental velocity determinations are used, the solution yields a separation point at about 82 degrees. The observed separation point is

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[2]For a complete presentation of this method, see [2], pages 154-162.

[3]The boundary layer separation point is defined as: $\partial u/\partial y|_{y=0}$
where u is the fluid velocity parallel to the body surface,
and y is directed normal to the surface, with its origin at the surface.

at about 81 degrees. Herein then lies the problem with the above practice: even at high Reynolds numbers there is sufficient interaction between the boundary layer and the external "inviscid" flow to cause considerable error in the results of essentially exact methods of solution. As will be shown in Section III, the separation point is critical in any method that attempts to mathematically generate a reasonably correct drag coefficient curve.

In recent years, extensive experimental work with the flow past bluff bodies has provided researchers with a clearer understanding of the mechanism of boundary layer separation. As a result, several models have been devised to portray the overall fluid action. However, to date none has provided the sought-after general solution.[4] A survey of the literature indicates that there are two basically different analytical approaches being made. The first, and probably the most obvious, is the solution of the nonlinear Navier-Stokes equations using finite difference techniques. The second is a potential flow model in which the shear layers are represented as point concentrated vorticies. Both these models, particularly the latter, will be described in greater detail in succeeding sections.

Since this thesis does not propose a general solution, it is hoped that the following sections will help to clarify the problem and to provide a basis from which future work can be done. Section II reviews the literature on separated flows; concentrating on those aspects that relate to incompressible flow about circular cylinders. Section III presents the two potential flow models that the author used in an attempt to generate the aforementioned drag curves.

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[4] The development of the high speed, large storage capacity digital computer has also been extremely important to the study.

II. REVIEW OF THE PRESENT STATE OF KNOWLEDGE

Because of the large number of references available on separated flows, the author has elected to limit this review through the use of the following guidelines. First, since there still exists a lack of basic understanding of the mechanism of separation only references to studies involving essentially incompressible fluid will be used. This enables the added complications of compressible phenomenon, both qualitative and mathematical, to be ignored. Secondly, a study of the literature reveals that several different geometries have been used both in experimental and analytical work; the results not always consistent nor related to each other. Therefore, the circular cylinder is taken as the basic geometry for this review. The use of a circular cylinder implies, to a large extent, two-dimensionality; which is another simplification. Though particularly helpful from a qualitative standpoint, this simplification is also useful mathematically. Thirdly, the information will not necessarily be presented in chronological order as is so often done in reviews of this nature. It is felt that a loose division by Reynolds number is more pertinent since the analytical solutions thus far derived are only applicable to given ranges of this "magic number," as von Kármán calls it. Lastly, the discussion will in general be limited to the laminar flow regime. As the Reynolds number increases above the critical value, increasing consideration must be given to compressibility and eventually to cavitation; two complicating subjects outside the scope of this thesis.

A. REYNOLDS NUMBER LESS THAN ONE

In this Reynolds number range, the viscous forces acting on the

cylinder are considerably larger than the inertial forces. This leads to the conclusion that if the inertia terms are dropped from the Navier-Stokes equations, an approximate solution may result that would compare favorably with experimental data. Stokes, as previously indicated, did this in 1851 when he presented the first solution for the drag on any body. Though he used a sphere in his solution, it is still worthwhile presenting here. The mathematical details are not the same, but the basic steps are.

The complete Navier-Stokes equations for incompressible, constant temperature conditions are (expressed in three-dimensional Cartesian tensor notation [4]):

$$\frac{dq_i}{dt} = f_i - \frac{1}{\rho} P_{,i} + \nu q_{i,jj} \quad . \tag{4}$$

The incompressible continuity equation is:

$$q_{i,i} = 0 \quad . \tag{5}$$

By dropping the inertia and body force terms, then cancelling density from both sides of the rearranged equation, one has

$$P_{,i} = \mu q_{i,jj} \quad . \tag{6}$$

The four equations represented by (5) and (6) have the same usual boundary conditions used for any boundary layer problem of this nature when the body is immersed in an infinite fluid: the fluid velocities normal to and tangent to the body surface are equal to zero.

After taking the divergence of both sides of equation (6), it becomes:

$$P_{,ii} = \mu q_{i,ijj} \quad . \tag{7}$$

But the right hand side of equation (7) can be written as $\mu(q_{i,i})_{,jj}$ which is the second derivative of the continuity equation and therefore equal to zero. This means that in creeping motion, the pressure field satisfies Laplace's equation and hence is a potential or harmonic function. Now, by introducing the two-dimensional stream function,

$$q_{,i} = -\epsilon_{ij3}\,\psi_{,j} \quad , \tag{8}$$

into the right hand side of equation (7), dropping $P_{,ii}$ and μ the two-dimensional biharmonic equation results.

$$D^4\psi = D^2(D^2\psi) \tag{9}$$

Without going into the mathematical details, it is sufficient to say that the pressure and velocity components reduce to:

$$\begin{aligned}
u &= U_\infty\left[\frac{3}{4}\frac{c\,x^3}{r^3}\left(\frac{c^2}{r^2}-1\right)-\frac{1}{4}\frac{c}{r}\left(3+\frac{c^2}{r^2}\right)+1\right] \\
v &= U_\infty\frac{3}{4}\frac{c\,x\,y}{r^3}\left(\frac{c^2}{r^2}-1\right) \\
w &= U_\infty\frac{3}{4}\frac{c\,x\,z}{r^3}\left(\frac{c^2}{r^2}-1\right) \\
P-P_\infty &= -\frac{3}{2}\frac{\mu\,U_\infty\,c\,x}{r^3} \\
r^2 &= x^2+y^2+z^2
\end{aligned} \tag{10}$$

The geometric configuration is such that the center of the sphere is at

the origin of the coordinate system and is placed in a positive velocity field, U_∞ , which is parallel to the x-axis.

Next, by using the standard definitions of shear stress along with equation (10), the viscous and pressure forces can be found by integration over the sphere's surface. The total drag for the sphere then becomes,

$$F_D = 6\pi\mu c U_\infty \quad . \tag{11}$$

If a drag coefficient is formed in the normal manner (i.e., referring to the dynamic head and frontal area), then,

$$F_D = C_D \pi c^2 \left(\frac{1}{2}\rho U_\infty^2\right) \quad . \tag{12}$$

Equating (11) and (12), then,

$$C_D = \frac{24}{Re} \tag{13}$$

Comparison of this result with experimental data indicates that these equations are valid only for cases where $Re < 1$. A classic case of application is that of small, light particles, assumed spherical in shape, settling in a fluid. Assuming that the particle is at its terminal velocity, then summing the forces yields,

$$\frac{2}{9} c^2 g \left(\rho_{Sphere} - \rho_{FLUID}\right) = \mu U_\infty \tag{14}$$

Letting $\Delta\gamma = g(\rho_{\text{sphere}} - \rho_{\text{fluid}})$ then equation (14) reduces to

$$\frac{c^2 \Delta\gamma}{\mu U_\infty} = \frac{9}{2} \quad , \tag{15}$$

which is the famous Stokes' number.

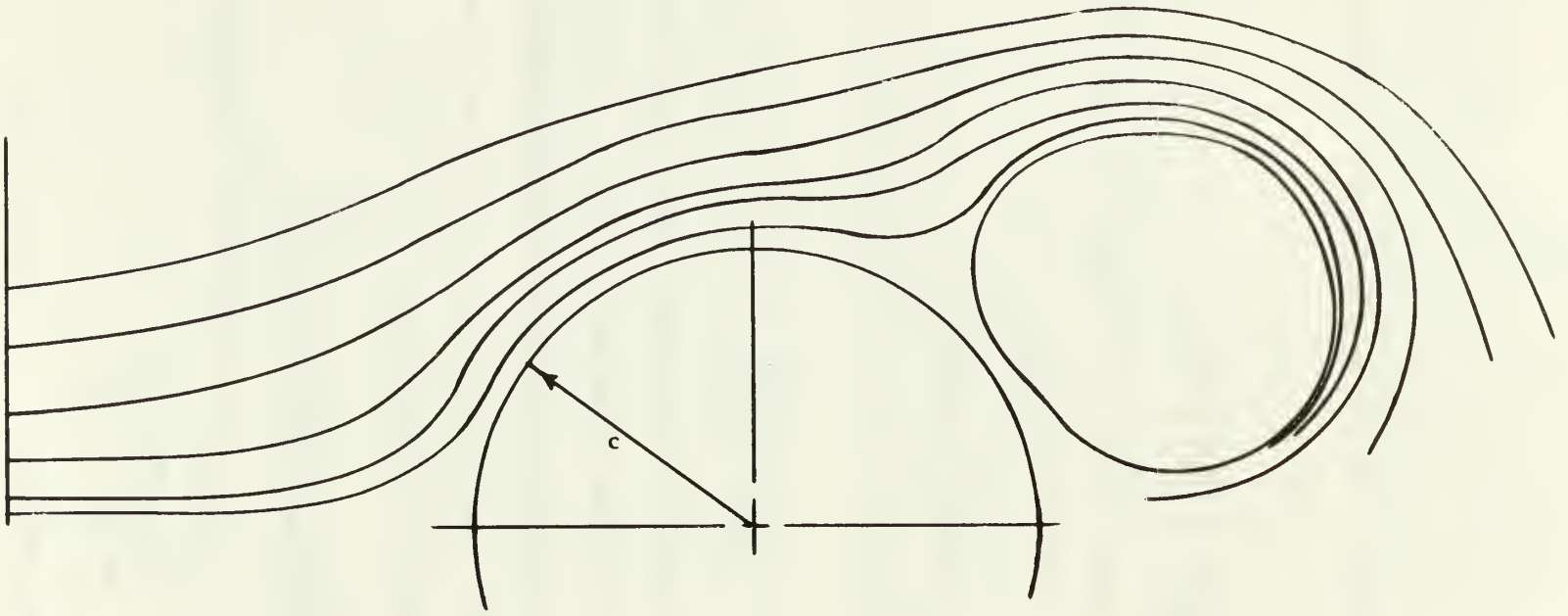

Figure 5. Flow Line Formation Around a Circular Cylinder for $3 < Re < 40$. Bottom is mirror image of this.

C. W. Oseen, in a paper published in 1910, improved on the above by taking the inertia terms of the Navier-Stokes equations partly into account [2]. By doing so, the resultant drag coefficient became:

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16} Re\right) . \qquad (16)$$

Comparison of (16) with experimental data shows favorable correlation up to a Reynolds number of about 5.

In recent years, other investigators have formulated other solutions, however, their main contributions have not necessarily been to engineering but more to mathematical prowess. Kaplun [5] based his on an asymptotic solution of a series expansion of the Navier-Stokes equations. The results are generally restricted to two-dimensional bodies with equivalent Reynolds numbers less than one. Sir Horace Lamb [6] also presented a solution applicable to this range. He followed the basic ideas of Oseen, however, where Oseen used a sphere, Lamb used a circular cylinder.

B. REYNOLDS NUMBER BETWEEN THREE AND FORTY

In this Reynolds number region, the boundary layer is still laminar, however, separation occurs causing the formation of two vortex cores in the wake of the cylinder as indicated in Figure 3. This region is sometimes referred to as the "Twin-Vortex Stage." A great deal of experimental evidence indicates that the cores are generally stable in this region provided that no external excitation occurs. It is in this region also that the greatest successes have occurred with the "exact" solution of the Navier-Stokes equations. Remembering that, above a Reynolds number of about one, the inertia forces are too large to neglect, the phenomenon

requires the solution of the full Navier-Stokes equations.[5] This is most readily done through the use of either the forward or backward finite difference techniques. Until recently, without the advent of the digital computer, these methods have been difficult to employ. Thom [7] in 1933 used a relaxation method of hand calculation to produce the streamline pattern shown in Figure 4 at a Reynolds number of 20.

It is true that the relaxation method can be programmed for a digital computer and made to converge to any realistic degree of accuracy, however, it is also true that computation time increases rapidly for a slight increase in desired accuracy. Thus this method is not considered the most desirable. The Gauss-Siedel iteration method is somewhat different in procedure, but can require the same amount of computer time for solution. It should be noted that both of these methods can be modified if done by hand since one can make shortcuts that are difficult to program for the computer. Even so, the digital computer reduces the time required from ".....about one year and a half with twenty working hours every week, with a considerable amount of labour and endurance ....."[8] to a total computation time of about three to five hours. Even this could probably be reduced by streamlining the iterative procedures. Robertson [1, p. 366] summarizes these methods quite well.

Though these analytical methods produce interesting flow patterns, their primary reason for use is to give needed information about separated flow. Unfortunately, they do not appear to explain the mechanism of separation or more importantly the feedback aspects of the wake on

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[5] Body forces, f_i, are usually neglected in these solutions since their presence adds nothing to the understanding of the problem and in general add unwanted complications to the mathematics.

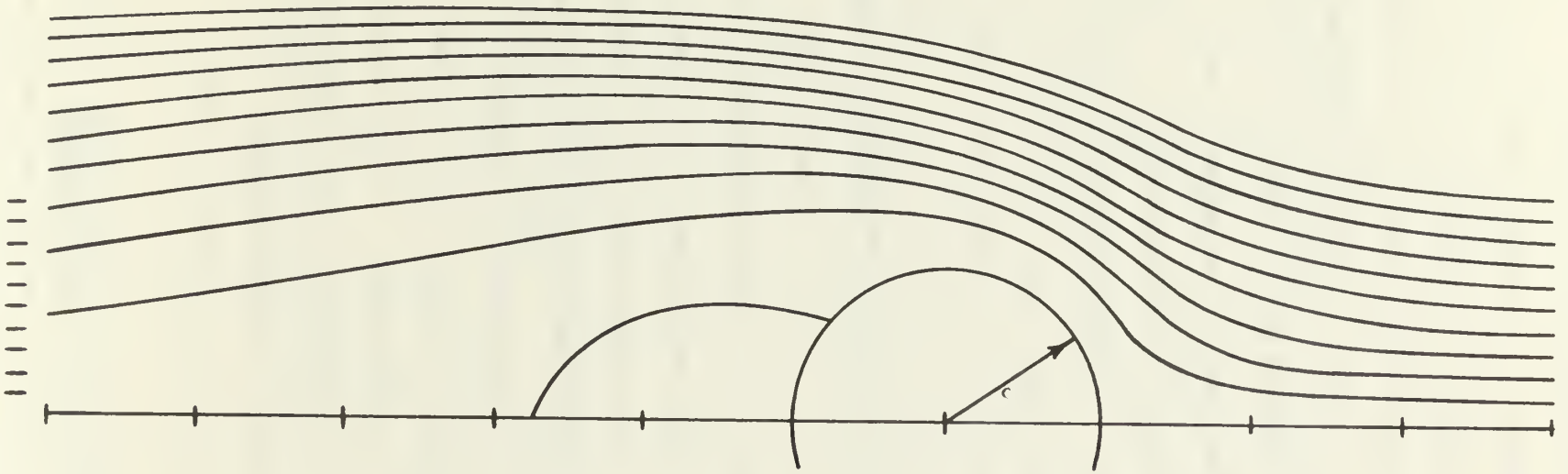

Figure 4 Calculated Streamlines past a Circular Cylinder at Re = 20, by Thom (1933) using Relaxation Method. The Undisturbed Flow Streamlines are Represented by the short lines at the end.

the vortex generation in the boundary layer. The experimental and analytical work does yield approximate Reynolds numbers for the beginnings of separation and the formation of the Kármán vortex street; about 5 and 40 respectively. From an analytical standpoint, the mesh size of the finite difference grid and the method of defining the conditions along the outside grid points[6] can affect the resultant separation and wake. Experimentally, just the placement of the confining walls of the apparatus used for testing can increase the Reynolds number, at which the wake becomes unstable, by more than 300 percent [9]. It should be noted here that although the wake is generally considered to be stable for steady flows in the Reynolds number range under discussion, it can be made unstable both theoretically and experimentally under artificial excitation. The ease with which this can be done, of course, increases as Re approaches the range of instability, making exact determination of the Reynolds number at which the vortex shedding begins very difficult. This same idea holds true for the determination of the separation Reynolds number. In both cases there appears to be some experimental evidence to indicate that the Reynolds number is not the exact similarity parameter that should be used in comparing results; the reasons for this are not entirely known. It is the belief of the author, however, that these disparities in results are probably due to variance in technique rather than to any fault of the parameter. It is recognized that in actuality there may be a three-dimensional effect that has not been taken into consideration theoretically or experimentally. Herein may be much of the reason for the differences in experimental results.

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[6] Requiring that the velocity on the surface of the cylinder be zero provides the boundary condition for the grid points on the cylinder surface.

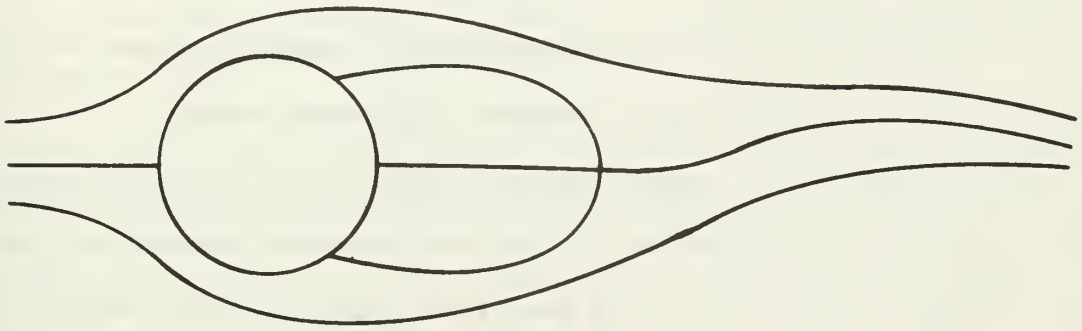

Figure 5. Beginnings of Vortex Street, Re $\approx$ 40.

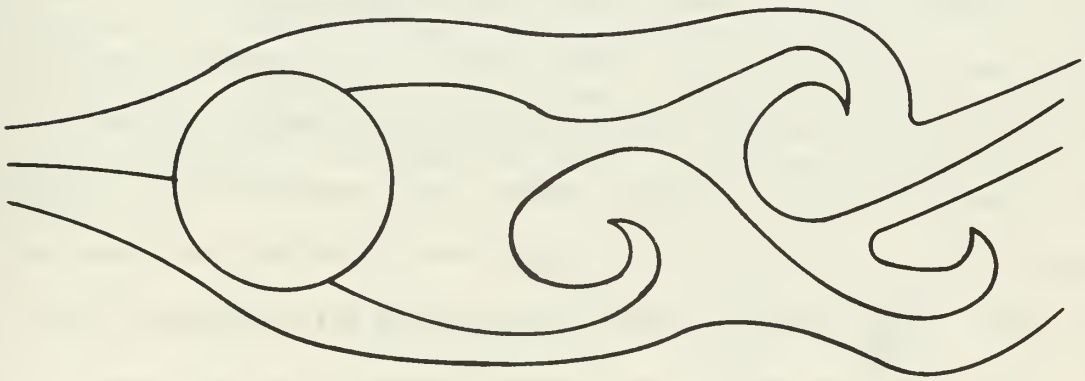

Figure 6. Karman Vortex Street Indicating Cyclic Shedding Vortex Cores, $80 < Re < 300$.

C. REYNOLDS NUMBER BETWEEN FORTY AND THREE HUNDRED

The primary phenomenon represented in this region is the Kármán vortex street; from the condition of an almost stable wake with slow viscous decay to the ultimate condition of cyclic detachment of vortex cores. Figures 5 and 6 indicate the flow patterns at these two "extremes." This region is also characterized by a smooth decrease in drag coefficient and an almost linear increase of Strouhal number, S.

Though study in this region has not produced any startling insight into the mechanism of separation, it has provided much information about vortex streets. Basically, it has been found that given two parallel vortex sheets separated by some initial distance, a solid body need not be present for a vortex street to develop [10]. The creation of these sheets is of course dependent on the viscous boundary layer effects caused by the presence of the solid body.

Due to the oscillatory nature of the vortex core separation into the wake, there is a cyclic shift in the separation points of the boundary layers on either side of the cylinder. This action is evidence of the strong "feedback" effect that the wake conditions have on the boundary layer in this Reynolds number region. This condition exists during the so-called steady state period after the various parameters such as drag coefficient, Strouhal number, etc., have settled to fairly constant values. In impulsively started flow situations, the flow patterns above and below the cylinder are basically mirror images of each other. The mechanism that causes eventual asymmetry is not clearly understood. In recent years, with the help of the digital computer, this asymmetry has been generated in finite difference solutions by the artificial insemination of small amounts of extra vorticity into one of the boundary layer separation regions, once the twin core pattern was

fairly well developed. Using this method, Fromm and Harlow [11] were able to duplicate the flow patterns around rectangular cylinders at these intermediate Reynolds numbers. At the lower numbers the system "absorbed" the extra vorticity with only minor disturbances to the wake, while at higher Reynolds numbers, the vortex cores separated cyclicly to form a vortex street.

Though the details of solution of the Navier-Stokes equations varies with the investigator, the same general procedures are used by all. The following is an attempt to clarify the salient points involved, but first, the reason for using these equations should be reiterated. Through this Reynolds number region, there appears to be a balance of sorts between the viscous and inertia forces, or effects, which implies that neither can be neglected or ignored. In the potential model that will be discussed later, the viscous forces are neglected from a drag standpoint. In the Stokes solution for $Re < 1$, the inertia forces were neglected with no apparent loss of practical accuracy. This region then appears to be nature's transition region from one extreme to the next, and therefore the complete governing equations must be used.

The method, in general, is as follows; dropping the body force term from equation (4), and expanding to the two-dimensional case, there results:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (17)$$

and,

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right). \quad (18)$$

Eliminating pressure by cross-differentiation and subtraction, one has:[7]

$$\frac{\partial}{\partial t}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+\frac{\partial u}{\partial x}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+\frac{\partial v}{\partial y}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right] \tag{19}$$

$$+u\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+v\frac{\partial}{\partial y}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]$$

$$=\nu\left\{\frac{\partial^2}{\partial x^2}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+\frac{\partial^2}{\partial y^2}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]\right\}$$

The vorticity vector is defined as

$$\zeta=\epsilon_{ijk}\,q_{k,i} \tag{20}$$

which means that since only two dimensions are involved, vorticity exists only parallel to the Z-axis. Therefore, letting

$$\zeta=\zeta_3=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{21}$$

and substituting back into equation (19), one has

$$\frac{\partial \zeta}{\partial t}+\zeta\frac{\partial u}{\partial x}+\zeta\frac{\partial v}{\partial y}+u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}=\nu\left[\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}\right],$$

or finally,

$$\frac{\partial \zeta}{\partial t}+\frac{\partial u\zeta}{\partial x}+\frac{\partial v\zeta}{\partial y}=\nu\left[\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}\right]. \tag{22}$$

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[7] u, v, and P must be assumed continuous functions of x, y, and t.

The system of equations needed becomes complete by introducing the stream function relationship,

$$q_i = -\epsilon_{ij3}\,\psi_{,j} \qquad (8)$$

Repeat into the continuity equation, $q_{i,i} = 0$. For the irrotational case, this would result with the Laplacian of the stream function equal to zero. However, there is rotation present and this introduction must be handled differently. By adding ζ to both sides of the continuity equation, one obtains

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta \,. \qquad (23)$$

The rearranging of the left side produces;

$$\frac{\partial}{\partial x}[u+v] + \frac{\partial}{\partial y}[v-u] = \zeta \,. \qquad (24)$$

Now substituting for u and v the appropriate expanded relation from equation (8), i.e., $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, and then performing the partial differentiation as indicated in front of each bracket, the equation becomes:

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y^2} = \zeta \,.$$

This of course reduces to a form of Poisson's equation,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\zeta \,. \qquad (25)$$

Equation (22), when written in terms of ψ and ζ reduces to

$$\frac{\partial \zeta}{\partial t} + \left[\frac{\partial \psi}{\partial y}\frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \zeta}{\partial y}\right] = \nu\left[\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right], \qquad (26)$$

which is somewhat easier to program since the necessity of finding the velocity gradients has been eliminated. If desired, these could be found from the other forms of the equations given above. As before, the assumption concerning continuous functions noted in footnote 7 still applies.

The specific finite difference formulas used to convert equations (25) and (26) into a numerically solvable system depends basically on the largest allowable truncation error and on the mesh size of the grid points. Crandall [12] discusses quite thoroughly all the aspects involved for both these considerations. It is sufficient to say that greater accuracy requires less truncation error and a finer mesh size. Generally, since finer mesh sizes require larger computer storage capabilities, they are used only in the areas of the flow field where large gradients are expected.

There are in general two basic methods of solving this system of equations. The one most widely used, until recently, has been of an iterative nature such as the relaxation or Gauss-Siedel methods; or modifications thereto. The primary concern here is that of convergence and stability of solution. The Gauss-Siedel method is adversely susceptible to both of these while the relaxation method generally does not have a stability problem, but can require an abnormally large number of iterations before some degree of convergence is attained. These methods are however, fairly simple to program. With the advent of the digital computer, more sophisticated numerical methods can be used to solve the system of equations explicitly using matrices. Methods such as Gaussian elimination with back substitution can produce a very accurate inverse for a matrix that is not singular, and a properly

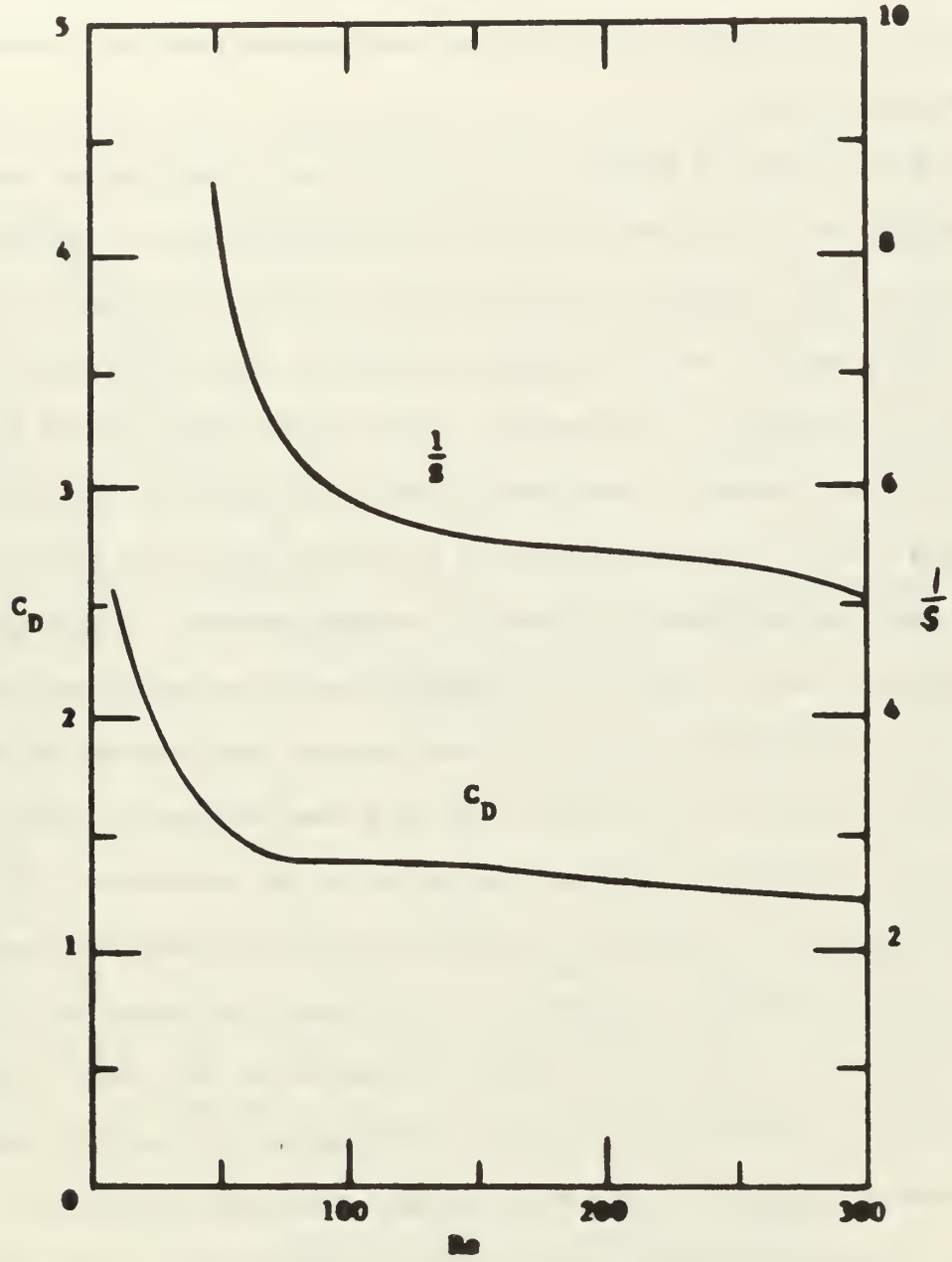

Figure 7. Mean Drag Coefficient and Inverse of Strouhal Number Versus Reynolds Number

constituted matrix representing a physical system should not be singular. For large systems of equations, the coefficient matrix can be reduced to a triangular form through the use of a sequence of linear operations. The inverse of this reduced matrix is then used rather than the inverse of the complete system.

Besides Fromm and Harlow [11] several other investigators have studied this Reynolds number region using modifications of the above procedures. Most numerical solutions have involved rectangular cylinders due to the ease with which Cartesian coordinate finite difference equations can be applied. Thompson [13] used a Gauss-Seidel method but appears to have assured convergence through the use of an "acceleration parameter" which is determined from a consideration of the eigenvalues of the matrices involved. His results compare favorably with experimental data, however, some of his computer runs required as much as nine hours on an IBM 360/40. This could have serious implications as far as accuracy is concerned, since each time step must adequately converge before the conditions for the next time step can be calculated. If convergence presents no problem, then there is another important result of his work that should be mentioned: no artificial introduction of vorticity is required to cause the eventual formation of the vortex street.

Wang [14] used cylindrical polar coordinates in a matched asymptotic expansion solution of the Navier-Stokes equations. Though the results offer no new insight, the method is indicative of the mathematical attempts being made. Ingham's paper [15] offers constructive criticism of Payne's method of finite difference solution [16], and is indicative of the pitfalls waiting for the unwary. Ingham's examples of variation of calculated C_D for different mesh sizes at the same Reynolds numer is quite dramatic.

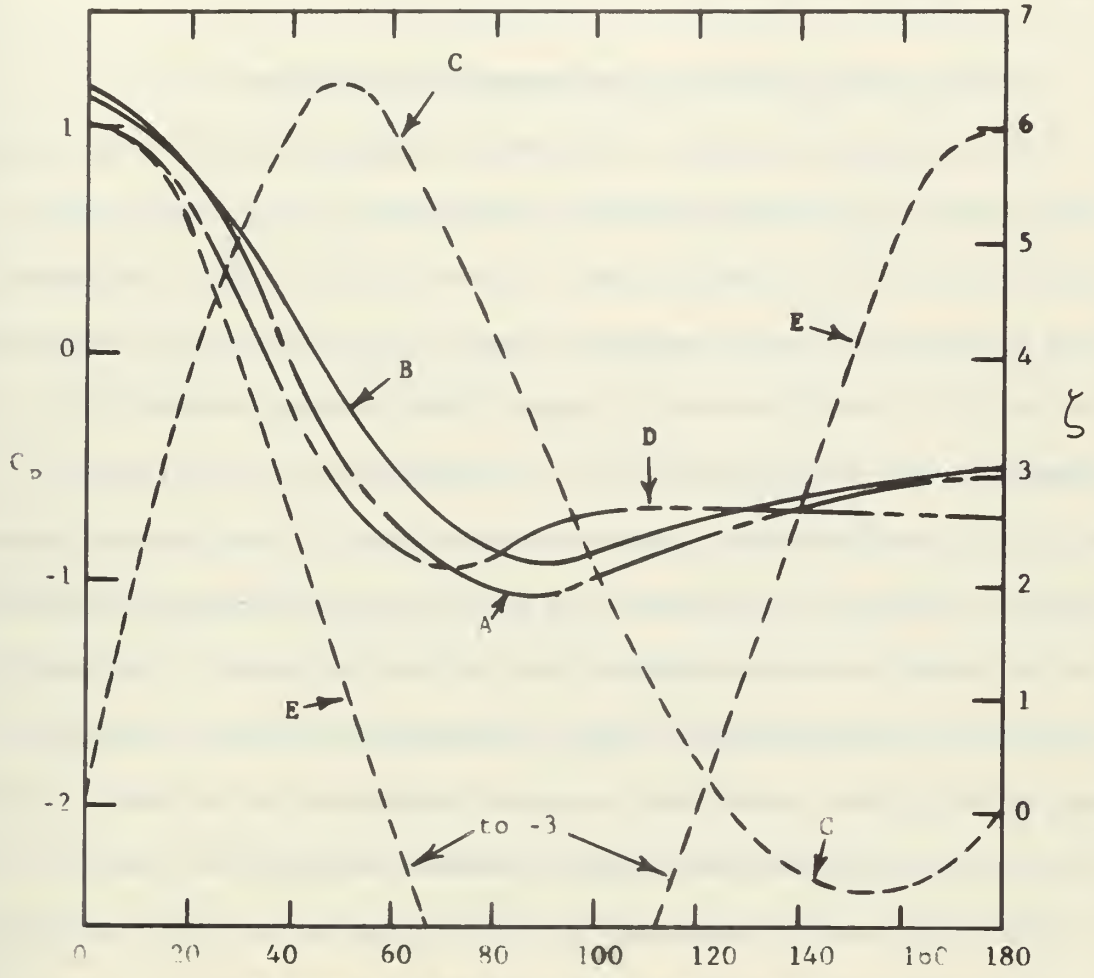

Figure 8. Experimental and Computed Pressure Coefficient and Computed Surface Vorticity (ζ) at Low Reynolds Numbers.

A Computed C_p, Apelt, Re = 40
B Computed C_p, Kawaguti, Re = 40
C Computed ζ, Apelt, Re = 40
D Experimental, Linke, Re = 2800
E Computed C_p, Potential Flow Theory

The pertinent experimental and analytical data derived by investigators in this Reynolds number range are shown in Figures 7 and 8. Important contributors of this information include Taneda [17], Tritton [18], Roshko [19], Rosenhead [20], Wille [21], and Apelt [22]; besides those previously mentioned.

D. REYNOLDS NUMBER BETWEEN THREE HUNDRED AND CRITICAL

This subcritical region is in general characterized by a relatively constant mean drag coefficient and a progressive loss in the distinct visualization of the Kármán street. As the Reynolds number increases there appears to be less "feedback" effect of the wake on the separating boundary layers which are still laminar. The Strouhal number has a tendency to decrease slightly with an increase of Re. It is in this region that the potential flow point vortex model of the feeding sheet appears to have the best possibility of providing information since the inertia forces are so much larger than the viscous forces. The use of the finite difference Navier-Stokes equations to provide a solution seems to break down due to the increased turbulence in the wake. Although a much finer mesh size might alleviate this problem, the fact that the equations now being used do not provide for velocity perturbation is probably the more basic reason. There appears to be no argument that the equations are applicable for use in describing the boundary layer on the front part of the cylinder almost up to the critical Re of about 5×10^5. As the Reynolds number increases from the low end of this region, the boundary layer effects appear to feed vorticity into the wake at an ever increasing rate causing the wake to become extremely turbulent. As this occurs, the well ordered vortex core regions of the Kármán street essentially disappear. Although the Strouhal number can

still be measured, the dominant periodicity seen in the lower Reynolds number wake flows all but disappears. Experimental studies using turbulent flow measuring techniques have confirmed the increasing turbulence with increasing Reynolds number. Ironically, the feedback problem diminishes, but the problem of transition from laminar to turbulent flow conditions becomes more important. The understanding of the latter problem is even less than that of the former. The end result is that the applicability of the laminar Navier-Stokes equations decreases with increasing Re.

In the "critical" Reynolds number region, loosely defined as $2 \times 10^5 < Re < 10^6$, there is a sharp decrease in the mean C_D down to about 0.3.[8] According to Schlichting [2, p. 39], this is due to the following reason: when the total drag on any body is primarily form drag, the transition from a laminar to a turbulent boundary layer causes a decrease in the size of the wake and results in a decrease in drag. This can be interpreted as saying that the drop in C_D is a boundary layer phenomenon, though it does not quite explain the entire mechanism. After this transition occurs, C_D increases to a value slightly lower than that before transition. There is also a return of a distinct vortex street. This time however, the "vortex cores" are highly turbulent internally. As the Reynolds number increases beyond this region, compressibility effects become greater, further complicating the problem. Depending on the fluid, very high Reynolds numbers can bring on the added, sometimes predominant, problem of cavitation.

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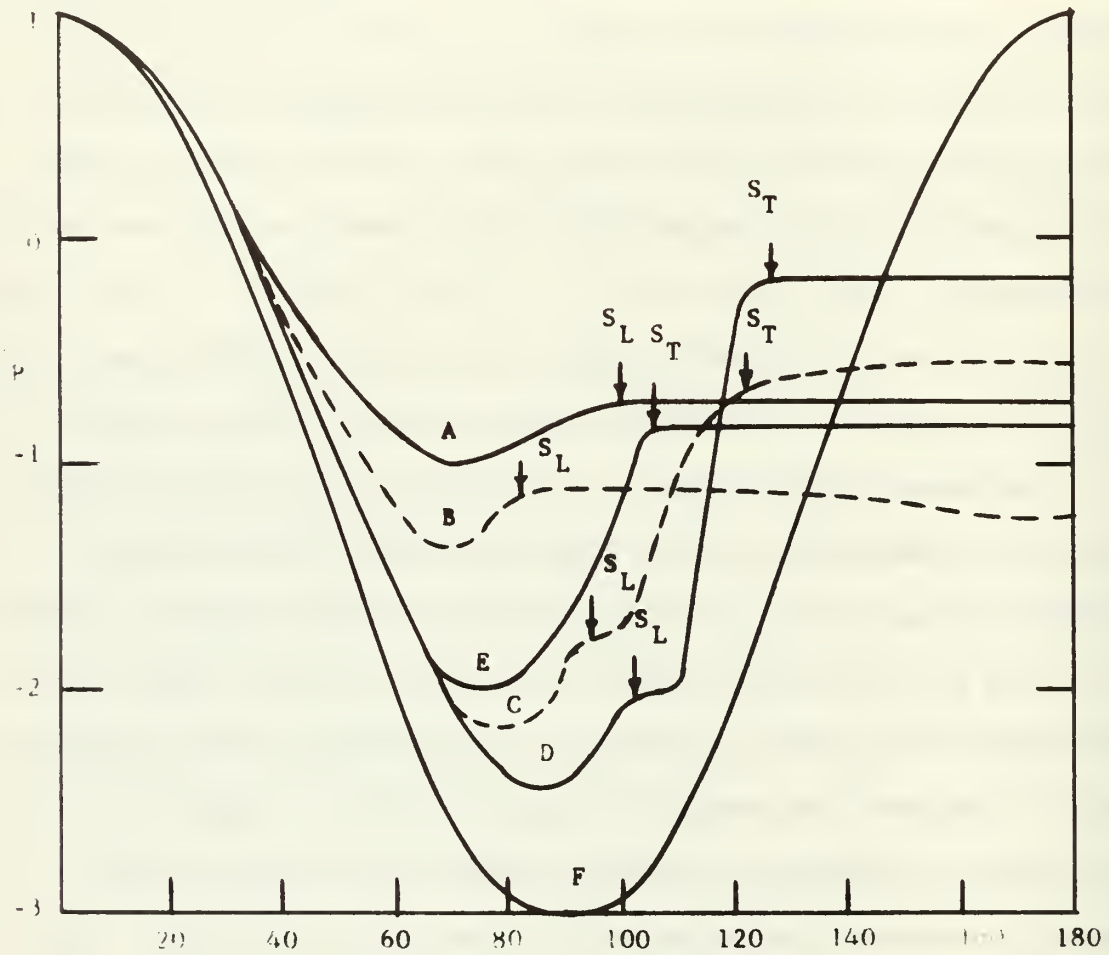
[8] This is true for a hydraulically smooth cylinder. The roughness of the cylinder surface can have a major effect on this minimum mean drag and the Reynolds number at which transition occurs. See [2, p. 621].



θ, Degrees measured from forward stagnation point.

Figure 9. Experimental Pressure Coefficients Found at Medium and High Reynolds Numbers Indicating Laminar (S_L) and Turbulent (S_T) Separation Points.

A Linke, Re = 2.8×10^3

B {Fage and} Re = 1.1×10^5
C {Falkner} Re = 2.1×10^5

D Flachsbart, Re = 6.7×10^5

E Roshko, Re = 8.4×10^6

F Potential Flow Theory

Experimental studies of the pressure distribution on the cylinder's surface have yielded some valuable information necessary for future analytical postulation. The pressure coefficient, defined as

$$C_p = \frac{2\Delta P}{\rho U_\infty^2} \tag{27}$$

is an excellent parameter for theoretical and experimental comparison particularly for creeping motions (Re $<$ 1), and at Reynolds numbers where the boundary layer separation points maintain fairly constant positions. At very low Reynolds numbers, the comparison is quite good especially on forward part of the cylinder, between points A and B in Figure 1. Between points B and C, the actual pressure decreases somewhat since energy is expended in the boundary layer; i.e., not all the kinetic energy acquired from A to B is recovered as potential energy as the fluid decelerates toward C. In Potential Flow Theory, points A and C are of course stagnation points, and whereas the pressure at A can be well approximated by this theory over the entire Reynolds number range, the pressure at point C cannot as indicated in Figure 9. Note how curve D, found for a Reynolds number above the critical, more nearly approximates curve F (potential flow). Photographs of the wake pattern produced for this post-critical Reynolds number shows a narrow wake with the rest of the flow field similar in appearance to that of an ideal fluid. Note also that after the boundary layer has separated, C_p assumes a nearly constant value, not the same for all Reynolds numbers however.

Though information about the Strouhal number is still incomplete above Re $\approx 10^5$, there is an interesting similarity between the inverse of this parameter and C_D. The curves for the two values, plotted over the whole range of Re from the beginning of core shedding through the

critical region, are very similar in shape. In fact, the similarity is so good that the drag coefficient could be written as

$$C_D = \frac{1}{S} - 1.4$$

with the resulting value having an accuracy within about 15 percent. The explanation for this result is not complete, however, the point here is that there is apparently a strong conceptual connection between core shedding frequency and drag. It is interesting to also note that through the region of nearly constant drag, not only is 1/S fairly constant, but that the oscillations of the boundary layer separation points are generally quite small in amplitude.

Investigators have attempted several approaches in the search for an analytic solution in this region and there have been some notable successes. Each approach, however, involves the use of at least one experimentally determined or arbitrarily selected parameter.

The free-streamline theory developed by Kirchoff, idealizes the free shear layers that separate from the cylinder by considering them to be surfaces of velocity discontinuity; hence a free streamline. This supposition works quite well for liquid jets discharging into a gas, however, for cylinder wakes, because of the assumption that the pressure in the wake is the same as the free stream pressure, the resultant calculated drag coefficients are much lower than those experimentally observed. The agreement can be greatly enhanced by the introduction of an arbitrarily selected parameter usually symbolized by K. This parameter is concerned with the depth of discontinuity (notch) in the logarithmic hodograph plane,[9] and is therefore concerned with the ratio between the

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[9] In this plane, the two-dimensional physical flow is represented by straight lines parallel to the two coordinate axes, and is the result of conformal mapping. See [1, p. 351].

free stream-line velocity and the free stream velocity. For very high Reynolds number flows, where cavitation exists, K is fixed by the cavitation index, σ. Fairly good agreement between experiment and theory exists in this case, particularly with respect to cavity flows.

Dennis and Shimshoni [26] have probably been two of the more successful investigators in analytically deriving the mean drag coefficient over the entire laminar boundary layer Reynolds number range. Their solution involves series expansions[10] of the Navier-Stokes equations, set up for finite difference evaluations. At slow, creeping motions, their solution converges to that of Stokes and Lamb, but at Reynolds numbers above about 1.5, the values of C_D are about 5 to 20 percent higher. They allow no asymmetry of vortex shedding. The mesh size they use appears to be much smaller than that of other attempts.

As noted earlier in the discussion, this Reynolds number region is characterized by the predominance of inertia forces over viscous forces. Also noted was that this leads to the possibility of using point concentrated, potential flow vortices to represent the shear layers or feeding sheets. Among those who have worked with this approach, Sarpkaya [35], Bryson [29],and Gerrard [25] have made some notable strides. The description of the work of the first two will be left to Section III, since it is basically with their ideas that the author worked. Gerrard's work, though similiar in mathematical approach to the others, appears to indicate a greater concern for wake characteristics than for finding solutions to the basic separation problem.

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[10] Similar to Oseen and Lamb.

Besides the basic points covered here, there is a great wealth of experimental information on the more detailed aspects of the subject that should be covered by any reader desiring to proceed further with the investigation. Toward this end, the author has attempted to compile the more notable of these in the List of References. Although some references have not been mentioned specifically herein thus far, this is not intended to mean they are of lesser value.

III. TWO ANALYTIC MODELS

A. INTRODUCTION

In the previous two sections, the discussion has centered on those few basic known facts and some of the many problems involved in this study. This section will present, in detail, two different analytic models, based on Potential Flow Theory, which are theoretically applicable to the relatively higher Reynolds region discussed in Section II (D).

The first model, called Model A hereafter, is conceptually more enlightening than the second; hence the reason for its presentation first. It is basically derived from Bryson's [29] model, but where his model was limited to the growth of axisymmetric vortex cores, the equations for Model A are recast to allow "forced" asymmetry to be included into the solution. To the author's knowledge, this is the first time this has been done. This method, like almost all the others, has no provision included in the governing equations for "natural" asymmetry. Thus if asymmetry is to be studied, its beginnings must be forced. In brief terms then, the basis of this model is that large vortex cores are fed vorticity through an infinitely narrow feeding sheet, or shear layer, connecting the center of the core with assumed stagnation points on the cylinder's surface.

In the second model, Model B, the shear layers are represented by a string of small strength vortices, called a vortex sheet; see [1, p. 111]. The equations for this model were also derived and programmed to allow forced asymmetry. Whereas Model A represents a continuously fed large vortex core, this one creates numerous small vortices which are

allowed to roll-up, coalesce and/or cancel at will. There are four birth areas in this model, two for the primary vortices, and two for the secondary vortices.[11] This arrangement is a modification of Sarpkaya's model [35] in which there are only two axisymmetric birth areas. Comparison between the two will be left until later in this section.

B. BASIC POTENTIAL FLOW EQUATIONS

The basic general equations used in both models will be presented in this subsection. The actual application will be left to the discussion of that particular model. It is not felt that the history behind these equations is relevant to this thesis since the interested reader can refer to Sarpkaya [36] for this information.

As mentioned earlier, in Section I, a two-dimensional flow system can be represented mathematically by equations written in complex variables. Since the mathematical development of this representation is quite lengthy, the following is only a brief part of what can be found in many reference books and papers on hydrodynamics. To start with, it is well to review the basic assumptions. First, the fluid is considered inviscid. This implies that there are no viscous forces present and therefore that the models derived should be applicable to the higher laminar Reynolds number range. Secondly, since this aforementioned Reynolds number range is generally devoid of compressibility effects, ρ will be assumed constant. The usual "steady flow" assumption

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[11] Vorticity is shed from the cylinder in the regions of boundary layer separation. These regions are associated with the "primary" boundary layers on the cylinder's upstream side and the "secondary" layers due to fluid back-flow on the downstream side. Since vorticity is created in these regions, they are called "birth" areas, and the small strength vortices created are called "nascent" vortices.

will not be made, because all the flow characteristics appear to be dependent on the manner in which the boundary layer generates vorticity and on the manner in which it diffuses into the wake. The key problem then is the formulation of a method by which vorticity can be generated in a supposedly inviscid fluid. This is in direct violation of Kelvin's theorem for inviscid fluids which says that vorticity once created, cannot be destroyed, and if not already existing, cannot be created. In order to overcome this, boundary layer information must be taken into consideration.

From basic considerations of hydrodynamics, an undefined number of vortices outside the boundaries of a circular cylinder, suspended in a uniform flow field as in Figure 10, can be represented by

$$W(z) = -U_\infty\left(z + \frac{c^2}{z}\right) + \frac{i}{2\pi}\left[\sum_{n=1}^{N} \Gamma_n \ln(z - z_n) - \sum_{n=1}^{N} \Gamma_n \ln\left(z - \frac{c^2}{\bar{z}_n}\right)\right]. \tag{28}$$

There are no symmetry restrictions on this equation. The normally excluded terms have been left out.[12] Next it can be shown that for a general location z, on or outside the cylinder, the fluid velocity is given by:

$$\frac{dW(z)}{dz} = -u + iv = U_\infty\left(\frac{c^2}{z^2} - 1\right) + \frac{i}{2\pi}\left[\sum_{n=1}^{N} \frac{\Gamma_n}{z - z_n} - \sum_{n=1}^{N} \frac{\Gamma_n}{z - \frac{c^2}{\bar{z}_n}}\right]. \tag{29}$$

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[12] These are the terms that are excluded by virtue of the condition that the vortices shed from the cylinder leave a circulation opposite to their own on the cylinder. See [38, p. 364].

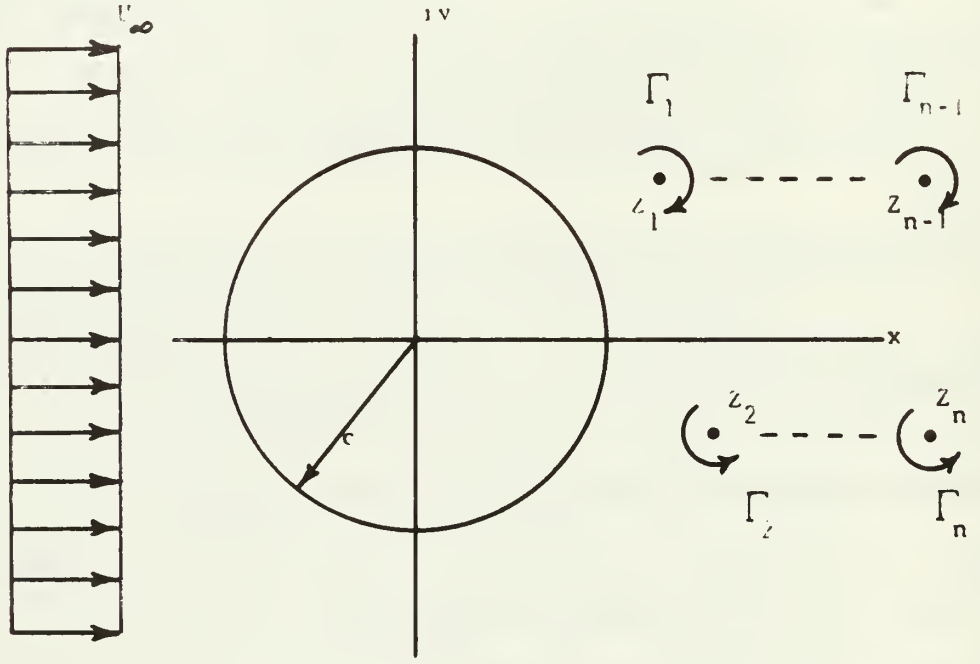

Figure 10. Circular Cylinder in the Presence of Numerous Vortices and a Uniform Flow Field.

Now that if $Z=Z_n$, the velocity goes to infinity. This is to be expected since the velocity at the center of an ideal, inviscid vortex is equal to infinity. This same equation can, however, be used to find the absolute velocity of the vortex with respect to cylinder, hereafter called the convective vortex velocity, by letting $Z=Z_n$ and disregarding the term that goes to infinity. This is due to the fact that the motion of a vortex is governed not by its own internal velocity distribution, but by the forces induced upon it by other entities in the flow field. See [1, p. 124].

Next, the drag and lift forces on a body can be calculated from the generalized Blasius' therorem,

$$\overline{F_D + iF_L} = \frac{i\rho}{2}\oint\left(\frac{dW(z)}{dz}\right)^2 dz + i\rho\frac{\partial}{\partial t}\oint W(z)dz \,. \qquad (30)$$

Writing

$$\overline{F_D + iF_L} = \overline{(X_1 - iY_1)} + (X_2 + iY_2) \,, \qquad (31)$$

equation (30) can be broken down into its so called "steady state" part,

$$X_1 - iY_1 = \frac{i\rho}{2}\oint\left(\frac{dW(z)}{dz}\right)^2 dz \,, \qquad (32)$$

and the time dependent part,

$$X_2 + iY_2 = i\rho\frac{\partial}{\partial t}\oint W(z)dz \,. \qquad (33)$$

The steady part, using equation (29), can be evaluated [35] such that:

$$X_1 + iY_1 = i\rho\sum_{n=1}^{N}\Gamma_n(u_n + iv_n) \,. \qquad (34)$$

The evaluation of equation (33) must wait until the discussion of the individual models since it involves some assumptions as far as the method by which Γ is to vary over the feeding zones. To continue, non-dimensionalizing can easily be done by referring all the length, velocity and time quantities to U_∞ and c; or for example,

$$\left.\begin{aligned} Z &= cZ' \,, \\ u + iv &= U_\infty (u' + iv') \,, \\ \Gamma' &= U_\infty c \Gamma'' \,, \end{aligned}\right| \qquad (35)$$

and,

$$t = \frac{ct'}{U_\infty}$$

Substituting equations (35) into (29); letting $U_\infty = 1.0$; $c = 1.0$, and dropping the primes for the sake of simplicity, this becomes;

$$-u + iv = \frac{1}{Z^2} - 1 + \frac{i}{2\pi}\left[\sum_{n=1}^{N} \frac{\Gamma_n}{Z - Z_n} - \sum_{n=1}^{N} \frac{\Gamma_n}{Z - \frac{1}{\bar{Z}_n}}\right] . \qquad (36)$$

Up to this point a discussion of image vortices has not been necessary, however, certain relations will be needed for future development of equation (33) in succeeding subsections. It can be shown by interpretation of Milne-Thompson's circle theorem [38, p. 157], or by using the method of images [38, p. 220], that the second summation in equation (29) and consequently in (36), are in reality "image" vortices inside the cylinder. The location of the image vortex, corresponding to the real vortex at Z_n is $c^2/\bar{Z}_n^2$. Its circulation is in the opposite sense as compared to that of its corresponding real vortex. Positive numerical values for circulation indicate rotation in the counterclockwise direction, while negative numerical values indicate clockwise rotation. In the derivation of the equations, the

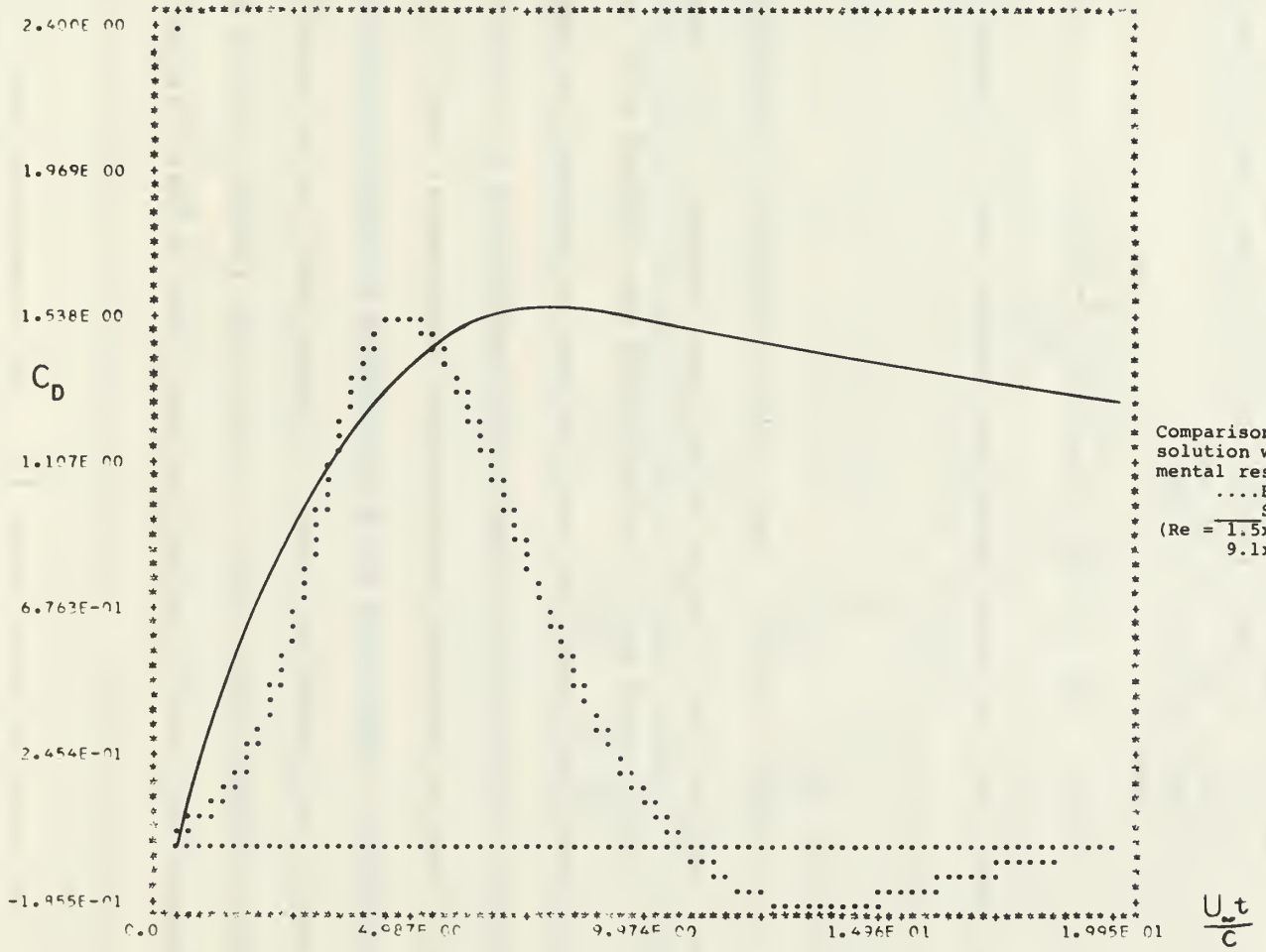

Figure 11

Comparison of Bryson's analytic solution with Sarpkaya's experimental results.
....Bryson [29]
____Sarpkaya [33]
(Re = 1.5×10^4 to 9.1×10^4, experimental)

symbol Γ has no inherent sign and can therefore take on positive or negative values as the situation warrants.

Since there are conditions well described in [38] that make equation (36) not applicable to the internal area of the cylinder, Sarpkaya [36] shows that the velocity of the image vortices are strictly a function of the position of the image vortex and the corresponding real vortex's velocity. By manipulating his expressions, the following equation for the image velocity results:

$$u_{in} + i\,v_{in} = \frac{-u_n + i\,v_n}{z_n^2} \qquad (37)$$

To reiterate, the term whose denominator equals zero is disregarded when solving for $-u_n + iv_n$.

C. MODEL A

As previously mentioned, this model is primarily a modification to Bryson's solution [29]. Though his solution appears to give reasonable drag coefficients during the beginning time periods of an impulsively started flow condition, once the peak is reached, the drag coefficient drops off to fluctuate about zero; Figure 11. This of course is not correct as Sarpkaya's superimposed experimental data [33] indicates. Though Bryson stops his solution when the vortices begin feeding circulation back to the cylinder,[13] this in itself is not enough to produce a reasonable drag curve. It was hoped that by studying his method, modified to allow asymmetry, more could be learned about the

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[13] The condition occurs when $\dot{\Gamma}_n$ (to be developed later) for a given vortex changes sign. If the solution is not stopped at this point a negative drag coefficient results as shown in Figure 11.

effect of vortex core shedding on the drag coefficient. Before a discussion of those results in undertaken, the equations must be developed.

Using Figure 12 as the basis for nomenclature definition, the equation development starts with the necessary modifications to equation (36). Though these equations can be constructed to allow more than two vortices, this will not be done at this time in order to show a simplified version of the technique. As will be indicated later, the full equations are obtained by the simple addition of a summation term to each of the following equations. So, based on two vortices, the velocity of vortex 1, if it were free,[14] is given by

$$-u_1 + iv_1 = \frac{1}{Z_1^2} - 1 + \frac{i}{2\pi}\left[\frac{\Gamma_2}{Z_1 - Z_2} - \frac{\Gamma_2}{Z_1 - \frac{1}{\bar{Z}_2}} - \frac{\Gamma_1}{Z_1 - \frac{1}{\bar{Z}_1}}\right]; \quad (38)$$

and likewise for vortex 2;

$$-u_2 + iv_2 = \frac{1}{Z_2^2} - 1 + \frac{i}{2\pi}\left[\frac{\Gamma_1}{Z_2 - Z_1} - \frac{\Gamma_1}{Z_2 - \frac{1}{\bar{Z}_1}} - \frac{\Gamma_2}{Z_2 - \frac{1}{\bar{Z}_2}}\right]. \quad (39)$$

Next, the feeding aspect of the vortex cores is handled by considering the vortex sheet, or feeding layer, as the combination of a single concentrated vortex, situated at the center of vorticity, with a connecting sheet to the body of vanishingly small vorticity. This sheet can now be considered as a mathematical branch line between Z_T and Z_1 for vortex 1, and Z_b and Z_2 for vortex 2. Since there is a discontinuity, conceptually, in ϕ across these branch lines, there is a velocity difference and therefore a pressure difference. This pressure difference

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[14] A free vortex is one that is no longer being fed circulation.

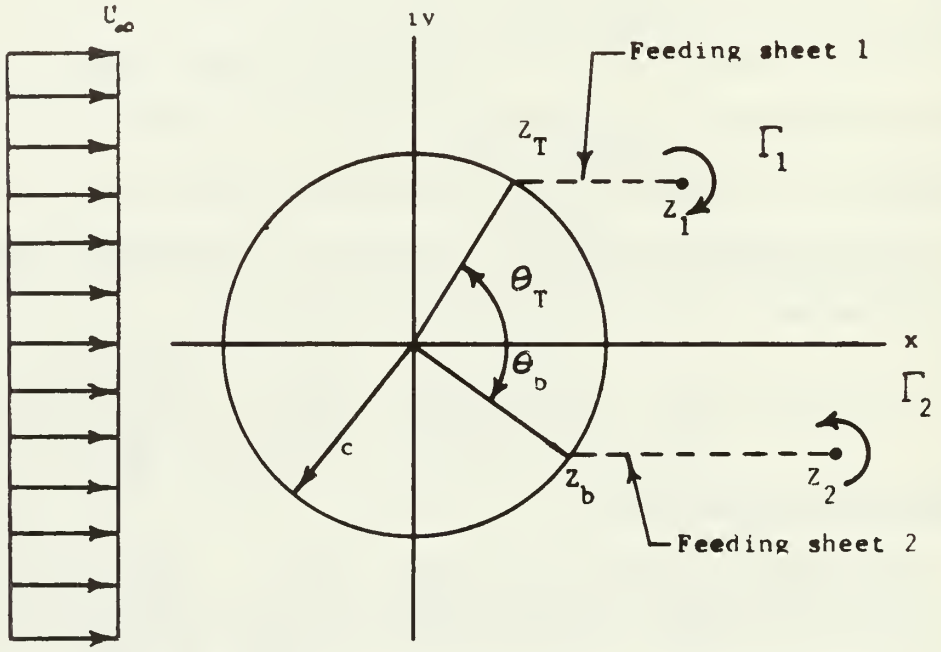

Figure 12. Model A, Indicating Capability for Asymmetry.

can be shown ([43] and [44]) to be equal to $\rho \dot{\Gamma}_n$. Since the distances over which the pressure differences act are $(z_1 - z_T)$ and $(z_2 - z_b)$ for vortices 1 and 2 respectively, the forces acting perpendicular to these sheets are given by $i\rho\dot{\Gamma}_1(z_1 - z_T)$ and $i\rho\dot{\Gamma}_2(z_2 - z_b)$. The forces acting directly on the vortices is given, for vortex 1, by

$$i\rho\Gamma_1\left(\overline{\frac{dW(z)}{dz}}\bigg|_{z=z_1} + \dot{z}_1\right),$$

and for vortex 2, by

$$i\rho\Gamma_2\left(\overline{\frac{dW(z)}{dz}}\bigg|_{z=z_2} + \dot{z}_2\right).$$

Thus to make the net forces on each sheet and its associated concentrated vortex vanish,[15] the sum of the respective forces must equal zero. Therefore, algebraically summing each pressure force with its proper vortex force, cancelling out i and ρ, and rearranging the terms, the two expressions formed become; for vortex 1:

$$\dot{z}_1 + (z_1 - z_T)\frac{\dot{\Gamma}_1}{\Gamma_1} = -\overline{\frac{dW(z)}{dz}}\bigg|_{z=z_1}, \qquad (40)$$

and for vortex 2;

$$\dot{z}_2 + (z_2 - z_b)\frac{\dot{\Gamma}_2}{\Gamma_2} = -\overline{\frac{dW(z)}{dz}}\bigg|_{z=z_2}. \qquad (41)$$

These equations indicate that unless $\dot{\Gamma}_n$ is zero, the nth vortex does not move with the local fluid velocity, $u_n + iv_n$, but with an "attached" velocity of $\dot{z}_n$.

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[15] The combination of vortex sheet and the concentrated vortex in a fluid medium has to be force free since no external force is exerted on the system.

$$\frac{\dot{\Gamma}_1}{2\pi} = \dot{\Gamma}_2 \frac{K_1 K_2}{2\pi} + \dot{Z}_1 K_5 K_{12} + \bar{\dot{Z}}_1 K_6 K_{12} + \dot{Z}_2 \frac{\Gamma_1}{2\pi} K_1 K_2$$
$$+ \bar{\dot{Z}}_2 \frac{\Gamma_2}{2\pi} K_1 K_{10} + K_1 \left(K_8 + \frac{K_T}{2\pi}\right)$$
$$\frac{\dot{\Gamma}_2}{2\pi} = \dot{\Gamma}_1 \frac{K_3 K_4}{2\pi} + \dot{Z}_1 \frac{\Gamma_1}{2\pi} K_3 K_{17} + \bar{\dot{Z}}_1 \frac{\Gamma_1}{2\pi} K_3 K_{16} + \dot{Z}_2 K_{13} K_{20} \qquad (43)$$
$$+ \bar{\dot{Z}}_2 K_{14} K_{20} + K_3 \left(K_{16} + \frac{K_b}{2\pi}\right)$$

where the constants K are defined by:

$$K_1 = \frac{(Z_T - Z_1)(Z_T \bar{Z}_1 - 1)}{1 - Z_1 \bar{Z}_1} \qquad K_5 = \left(\frac{Z_T \bar{Z}_1 - 1}{1 - Z_1 \bar{Z}_1}\right)^2$$

$$K_2 = \frac{Z_2 \bar{Z}_2 - 1}{(Z_T - Z_2)(Z_T \bar{Z}_2 - 1)} \qquad K_6 = \left(\frac{Z_T - Z_1}{1 - Z_1 \bar{Z}_1}\right)^2$$

$$K_3 = \frac{Z_b - Z_2)(Z_b \bar{Z}_2 - 1)}{(1 - Z_2 \bar{Z}_2)} \qquad K_7 = i U_\infty \left(1 - \frac{1}{Z_T^2}\right)$$

$$K_4 = \frac{(Z_1 \bar{Z}_1 - 1)}{(Z_b - Z_1)(Z_b \bar{Z}_1 - 1)} \qquad K_8 = i \dot{U}_\infty \left(1 - \frac{1}{Z_T^2}\right)$$

$$K_9 = \frac{1}{(Z_T - Z_2)^2}$$

$$K_{10} = \frac{1}{(Z_T \bar{Z}_2 - 1)^2}$$

$$K_{11} = \sum_{m=1}^{M} \Gamma_m \left[\frac{1}{Z_T - Z_m} - \frac{1}{Z_T - \frac{1}{\bar{Z}_m}} \right]$$

$$K_{12} = \frac{\Gamma_2}{2\pi} K_2 + \frac{K_{11}}{2\pi} + K_7$$

$$K_{13} = \left(\frac{Z_b \bar{Z}_2 - 1}{1 - Z_2 \bar{Z}_2} \right)^2$$

$$K_{14} = \left(\frac{Z_b - Z_2}{1 - Z_2 \bar{Z}_2} \right)^2$$

$$K_{15} = i U_\infty \left(1 - \frac{1}{Z_b^2} \right)$$

$$K_{16} = i \dot{U}_\infty \left(1 - \frac{1}{Z_b^2} \right)$$

$$K_{17} = \frac{1}{(Z_b - Z_1)^2}$$

$$K_{18} = \frac{1}{(Z_b \bar{Z}_1 - 1)^2}$$

$$K_{19} = \sum_{m=1}^{M} \Gamma_m \left[\frac{1}{Z_b - Z_m} - \frac{1}{Z_b - \frac{1}{\bar{Z}_m}} \right]$$

$$K_{20} = \frac{\Gamma_1}{2\pi} K_4 + \frac{K_{19}}{2\pi} + K_{15}$$

$$K_T = \sum_{m=1}^{M} \Gamma_m \left[\frac{\dot{z}_m}{(Z_T - Z_m)^2} + \frac{\bar{\dot{z}}_m}{(Z_T \bar{Z}_m - 1)^2} \right]$$

$$K_b = \sum_{m=1}^{M} \Gamma_m \left[\frac{\dot{z}_m}{(Z_b - Z_m)^2} + \frac{\bar{\dot{z}}_m}{(Z_T \bar{Z}_m - 1)^2} \right]$$

NOTE: Since the M vortices are unattached, $\dot{z}_m$ and $\bar{\dot{z}}_m$ are calculated from equation (29).

Since there are more unknowns than there are equations, the system is still ill-defined. Four more equations can be gotten by imposing the Kutta condition at the point from which the vortex cores are fed. This is done by assuming that the fluid velocity relative to the cylinder at these points is zero, i.e., Z_T and Z_b are stagnation points. By solving equation (36) for $-u_T + iv_T$ and for $-u_b + iv_b$, then setting these equations equal to zero and rearranging in matrix form, the two new equations become;

$$\frac{1}{2\pi}\begin{bmatrix} 1 & -K_1K_2 \\ -K_4K_3 & 1 \end{bmatrix}\begin{Bmatrix} \Gamma_1 \\ \Gamma_2 \end{Bmatrix} = \begin{Bmatrix} K_1\left(K_7 + \frac{K_{11}}{2\pi}\right) \\ K_3\left(K_{15} + \frac{K_{19}}{2\pi}\right) \end{Bmatrix} \tag{42}$$

The Ks are the same ones associated with equation (43). Though not included in equations (38) and (39) for the sake of simplicity, the above equations include the extra summation terms, K_{11} and K_{19}, necessary for the calculation if more than two vortex cores are present. To give the velocity equations (38) and (39) the same versatility, all that is necessary is to multiply these terms by $i/2\pi$, adding the first one to right hand side of (38) and the second one to (39). This has been done in the computer program for this model.

The next two equations are gotten by solving the first equation of (42) for Γ_1 and the second one for Γ'_2. Then the time derivatives, $\dot{\Gamma}_1$ and $\dot{\Gamma}_2$, are taken, assuming that Z_T and Z_b are constants. Since the procedure is long and involved, but quite straight forward, only the results will be presented; see equations (43). These equations also include the terms necessary for more than two vortex cores. There is also a provision made in these equations for $\dot{U}_\infty$ to be taken into consideration. This quantity was set to zero for the study undertaken in this

thesis, but could be utilized in future work. Note that the equations have been non-dimensionalized. This means that $\dot{U}_\infty$ is also a non-dimensionalized quantity.

This now leaves three six unknowns, Z_b, Z_T, Z_1, Z_2, $\bar{\dot{Z}}_1$, and $\bar{\dot{Z}}_2$ in the system of equations without a corresponding number of defining equations. The latter two unknowns can be easily handled by forming the conjugates of both sides of (40) and (41) to yield,

$$\bar{\dot{Z}}_1 + (\bar{Z}_1 - \bar{Z}_T)\frac{\dot{\Gamma}_1}{\Gamma_1} = -\left.\frac{dW(z)}{dz}\right|_{z=Z_1}, \qquad (44)$$

and

$$\bar{\dot{Z}}_2 + (\bar{Z}_2 - \bar{Z}_b)\frac{\dot{\Gamma}_1}{\Gamma_1} = -\left.\frac{dW(z)}{dz}\right|_{z=Z_2}.$$

as mentioned earlier, Z_b and Z_T are considered constants. This is an assumption based on experimental observations at higher laminar Reynolds numbers. The origins of these feeding sheets are in reality functions of time, but their motions are so small that the assumption seems justified. Besides, to consider $\dot{Z}_b$ and $\dot{Z}_T$ anything but zero, would have made the derivation of (43) at least another order of magnitude more tedious. This may, however, prove to be a necessary future sophistication of the model. As a result, these two locations are taken from experimental data to be about 50 degrees either side of the cylinder's downstream axis; i.e., θ_T = 50 and θ_b = -50 degrees.

No mention thus far has been made concerning the propagation aspects of this solution, however since time dependent solutions generally imply that starting and ending conditions are necessary, it is well to discuss these now with respect to Z_1 and Z_2. With the assignment of any values to these two parameters, there have been enough equations derived and assumptions made to have the proper balance between unknowns

and defining equations to describe the system at any given instant. Not just any value of Z_1 and Z_2 is desirable however, since this will not give even an approximate picture of core growth and drag buildup. Bryson [29], through certain mathematical considerations, found that the path line of the vortex cores, as they grow and move away from the cylinder, is unique in the sense that for any given feeding sheet origin, Z_T or Z_b, there can be only one path, per vortex, followed as time progresses. Without going into the details of his work, he also found that the feeding points, like the Föppl curve points,[16] are mathematical equilibrium positions for symmetric vortex pairs. On the basis of these findings, it is also determined that cores can leave the cylinder's surface only in a direction 30 degrees downstream from the tangent to the cylinder at the sheet origin. Bryson arbitrarily selected the starting values for Z_1 and Z_2 as a distance of 0.05c away from the origins along these respective preferred lines of departure. This practice was continued in this study even though the model can handle asymmetry and more than two vortex cores. It was felt that these are good assumptions for an initial study of asymmetry.

Except for determining when and how the vortex sheets are cut, there are no more assumptions to be made in completing the solution. The procedure for solving the previously developed equations and the development of the $(F_D + iF_L)$ equation are not complicated and follow next.

---

[16]The Föppl curve is a symmetric curve about the downstream X-axis originating at the rear stagnation point, on which a vortex pair can be symmetrically located such that they will maintain their positions. This is much the condition that exists at Reynolds numbers below about 50. See [6, p. 223].

$$
\begin{bmatrix}
\frac{1}{2\pi} & -\frac{K_1 K_2}{2\pi} & -K_5 K_{12} & -K_6 K_{12} & -\frac{\Gamma_2}{2\pi} K_1 K_9 & -\frac{\Gamma_2}{2\pi} K_1 K_{10} \\
-\frac{K_3 K_4}{2\pi} & \frac{1}{2\pi} & -\frac{\Gamma_1}{2\pi} K_3 K_{17} & -\frac{\Gamma_1}{2\pi} K_3 K_{18} & -K_{13} K_{20} & -K_{14} K_{20} \\
\frac{Z_1 - Z_I}{\Gamma_1} & 0 & 1 & 0 & 0 & 0 \\
\frac{\bar{Z}_1 - \bar{Z}_I}{\Gamma_1} & 0 & 0 & 1 & 0 & 0 \\
0 & \frac{Z_2 - Z_b}{\Gamma_2} & 0 & 0 & 1 & 0 \\
0 & \frac{\bar{Z}_2 - \bar{Z}_b}{\Gamma_2} & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix}
\dot{\Gamma}_1 \\ \dot{\Gamma}_2 \\ \dot{Z}_1 \\ \dot{\bar{Z}}_1 \\ \dot{Z}_2 \\ \dot{\bar{Z}}_2
\end{Bmatrix}
=
\begin{Bmatrix}
K_1 \left(K_8 + \frac{K_7}{2\pi} \right) \\
K_3 \left(K_{16} + \frac{K_6}{2\pi} \right) \\
u_1 + i v_1 \\
u_1 - i v_1 \\
u_2 + i v_2 \\
u_2 - i v_2
\end{Bmatrix}
\tag{45}
$$

Equations (40), (41), (43), and (44) can be rearranged and combined in a straight forward manner for the matrix equation (45). Both this matrix equation and equation (42) are solved using the partitioned matrix technique as given in Fox [39, p. 121]. Briefly, this involves breaking up the basic matrix equation into its real and imaginary parts and forming a new equation whose matrices are twice the order of the original. To describe the technique, suppose the original equation is;

$$\left[A\right] \left\{B\right\} = \left\{C\right\},$$

where [A] is a square matrix of order j, while $\left\{B\right\}$, and $\left\{C\right\}$ are column matrices with j elements each. Since all the elements of A, B, and C are assumed to be complex, the matrices can be written as;

$$\begin{aligned} \left[A\right] &= \left[A_R\right] + i\left[A_I\right], \\ \left\{B\right\} &= \left\{B_R\right\} + i\left\{B_I\right\}, \\ \left\{C\right\} &= \left\{C_R\right\} + i\left\{C_I\right\}. \end{aligned}$$

Substituting these expressions back into the original equation, results in

$$(\left[A_R\right] + i\left[A_I\right])\,(\left\{B_R\right\} + i\left\{B_I\right\}) = (\left\{C_R\right\} + i\left\{C_I\right\}).$$

Carrying out the indicated multiplication, and separating the real and the imaginary parts of both sides, yields

$$\left[A_R\right]\left\{B_R\right\} - \left[A_I\right]\left\{B_I\right\} = \left\{C_R\right\},$$

and,

$$\left[A_I\right]\left\{B_R\right\} + \left[A_R\right]\left\{B_I\right\} = \left\{C_I\right\}.$$

These two equations are combined to form the new matrix equation:

$$\begin{bmatrix} \left[A_R\right] & -\left[A_I\right] \\ \left[A_I\right] & \left[A_R\right] \end{bmatrix} \begin{Bmatrix} \left\{B_R\right\} \\ \left\{B_I\right\} \end{Bmatrix} = \begin{Bmatrix} \left\{C_R\right\} \\ \left\{C_I\right\} \end{Bmatrix}. \qquad (46)$$

Since there are no imaginary numbers involved, this equation is solved by existing numerical methods, i.e., the inverse of the coefficient matrix premultiplies the constant matrix to yield the unknown values of $\{B\}$. In the computer program for this model, the inverse is found using a standard inversion subroutine with row pivoting.

The propagation aspect of the solution is handled in the program through the use of a simple Runga-Kutta integration technique [40, p. 232] by finding the subsequent vortex positions, after a $\triangle t$ time step, from equations (40) and (41). There are, of course, more sophisticated techniques, but in view of the parameters assumed, it is felt that the increased accuracy is not necessary at this point. In fact, there is not really that much difference in results if a straight Eulerian approximation is used.

The final relation to be developed is that for $C_D + iC_L$. This is done by completing the derivation of equation (30), or more specifically by finding the appropriate solution for equation (33). For this case, in which the cores are fed by a single feeding sheet originating at a point, Sarpkaya, [33], and [35], has previously derived the necessary equation, so the details will not be given here. The result, in non-dimensional form and adapted to this model is:

$$C_D + iC_L = 2\pi\dot{U}_\infty + i\left[\dot{\Gamma}_1\left(Z_1 - \frac{1}{\bar{Z}_1}\right) + \dot{\Gamma}_2\left(Z_2 - \frac{1}{\bar{Z}_2}\right)\right]$$
$$+ i\left[\Gamma_1\left(\dot{\bar{Z}}_1 - \dot{\bar{Z}}_{i,1}\right) + \Gamma_2\left(\dot{\bar{Z}}_2 - \dot{\bar{Z}}_{i2}\right)\right] \qquad (47)$$
$$+ i\sum_{m=1}^{M}\Gamma_m\left[U_m + iV_m - (U_{im} + iV_{im})\right] .$$

The image velocities for the free and attached vortices are both calculated, in a sense, using equation (37). However, where the results of equation (29) are used directly for the vortices, the results of solving matrix equation (45) must be used for the attached vortices. The modification to equation (37), used for this purpose is; for the image of vortex 1,

$$\dot{Z}_{i1} = -\frac{\dot{\bar{Z}}_1}{\bar{Z}_1^2}$$

and for that of vortex 2,

$$\dot{Z}_{i2} = -\frac{\dot{\bar{Z}}_2}{\bar{Z}_2^2} \quad . \tag{48}$$

The computer program written for this model is listed in its entirety in the last portion of this thesis. It is written in FORTRAN IV as applied to the IBM 360/67 digital computer at the Naval Postgraduate School. In general, the calculations are carried out using sixteen digit numbers. The typical run, describing the impulsively started flow up to $\frac{U_\infty t}{c} \approx 15.0$, requires less than three minutes of computer execution time. Provided that the initial starting points, Z_1 and Z_2, are accurately defined, the solution appears to be stable and well behaved. The stability problem was previously discussed in connection with the vortex path line being unique. If the initial estimate for Z_1 and Z_2 are even slightly off, the solution technique attempts to return to the path, but as a consequence there may be some abnormally large absolute values for $\dot{\Gamma}_1$, $\dot{\Gamma}_2$, $\dot{Z}_1$, and $\dot{Z}_2$. To increase the stability, smaller time steps are taken at the start and just after a vortex has been shed and a new vortex starts to grow. The normal time step is 0.05, while this reduced time step is 0.02. The program will accept as many as twenty vortices before it terminates execution.

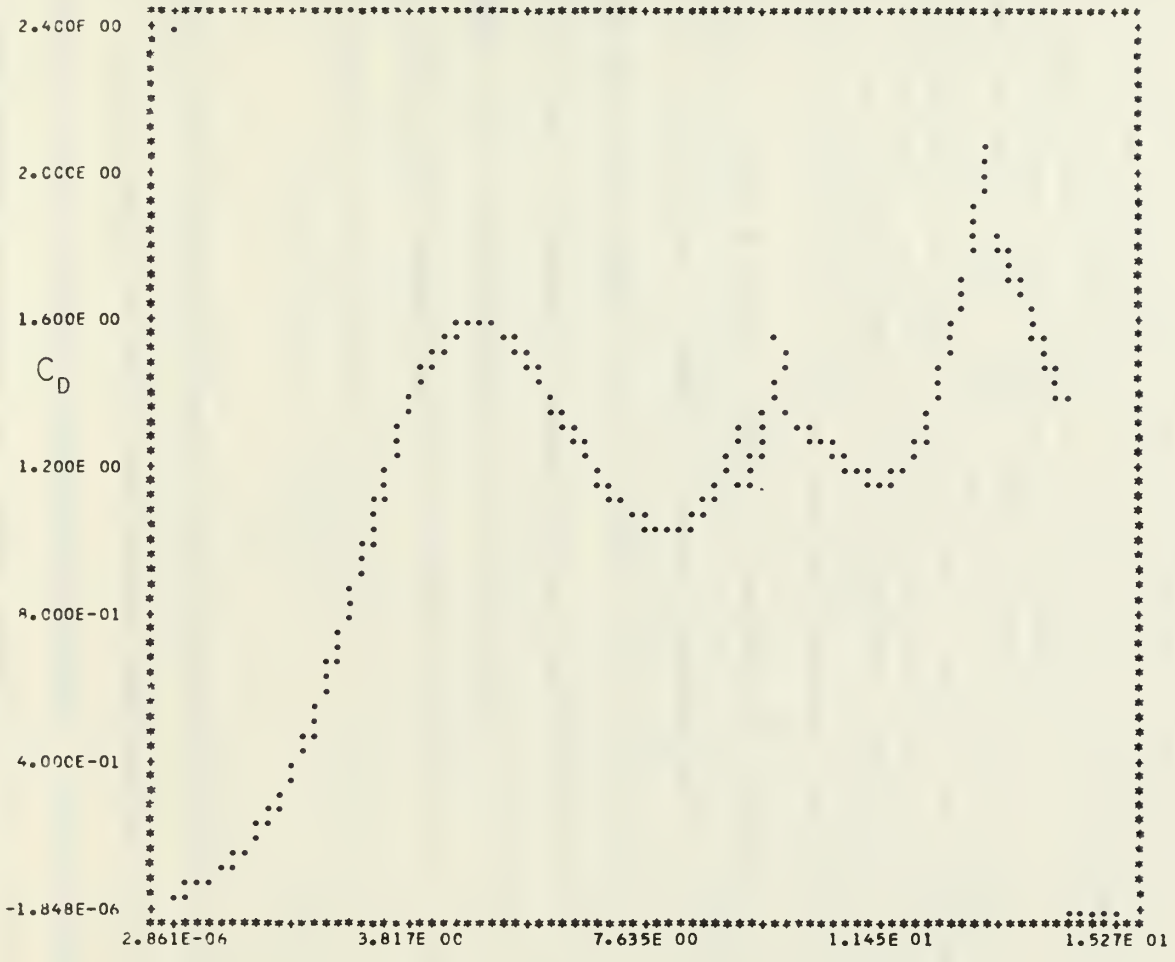

Figure 13

Drag coefficient curve calculated using Model A for an asymmetric case.

Since anyone desiring to utilize this program must become intimately familiar with its internal workings, it is felt that any further discussion is unnecessary here. There is a brief, but more detailed description in Appendix A.

It was possible, through the use of this program, to generate a reasonably good drag curve using asymmetric considerations. To start with, if no shedding was allowed, it was found that by multiplying the value of Γ_1, during each time step, from equation (42) by 1.05, prior to its use in equation (45), that the value of $\dot{\Gamma}_1$ never changed sign. It did however, peak at about the same $\frac{U_\infty t}{c}$ that it had previously (no asymmetry), but instead of decreasing to zero and subsequently changing sign; it decreased somewhat and then increased beyond bound. The drag curve followed somewhat the same pattern. By arbitrarily cutting the vortex sheet at the minimum $\dot{\Gamma}_n$ values, the drag curve in Figure 13 was generated. The computer listing for Model A, though meant to be modified for other uses, is in fact, the same program that produced this curve. It is realized that the amount of asymmetry imposed is much too large, but is an illustration of the possibilities. A good beginning point for future work, would be with von Kármán's stability conditions or to investigate the works done by Roshko [19] and Bearman [27].

D. MODEL B

Since this model is an attempt to approximate the vortex sheets generated by boundary layer separations, it is much more sensitive to the placement of the nascent vortices than is Model A to the location of the feeding sheet origins. Hence, the problem of mathematically describing vortex sheet formation is more obvious. This model has a distinct advantage over the other, in that it is not necessary to

incorporate any assumptions into the derivation of the basic equations of motion that take the place of matrix equation (45). The assumptions are present, however, in the determination of the strength and initial nascent vortex positions. Probably the most important single advantage of this model, is that in general, once the separation points and vortex strengths have been determined, the vortex sheets will roll up to form vortex cores, taking care of the vorticity transport and diffusion automatically. To discuss these aspects further will require the development of the applicable equations, so this follows next.

Since the nascent vortex is assumed to gain its total strength during the time step in which it is born, equation (29) can be used to determine its convective velocity. It is repeated here for convenience.

$$-u+iv=u_\infty\left(\frac{c^2}{z^2}-1\right)+\frac{i}{2\pi}\left[\sum_{n=1}^{N}\frac{\Gamma_n}{z-z_n}-\sum_{n=1}^{N}\frac{\Gamma_n}{z-\frac{c^2}{\bar{z}_n}}\right] \qquad \text{(29) Repeat}$$

The vortex convective velocity, as indicated in the initial discussion of equation (29), is calculated by setting $Z = Z_n$ and ignoring the term in the first summation that has the denominator equal to zero. The velocity of the image vortex can still be calculated from

$$u_{in}+iv_{in}=\frac{-u_n+iv_n}{\bar{z}_n^2} \quad . \qquad \text{(37) Repeat}$$

The drag and lift coefficients are again calculated from equation (30), however, this time, some liberties can be taken with its use. Again, since the nascent vortex is born during one time step, a part of the effect of the unsteady portion of (30), more specifically parts of equation (33), can be assumed negligible. In fact, a study of the results of [41] indicates that the algebraic sum of the values of the terms concerned with feeding and growth of the vortices actually only account

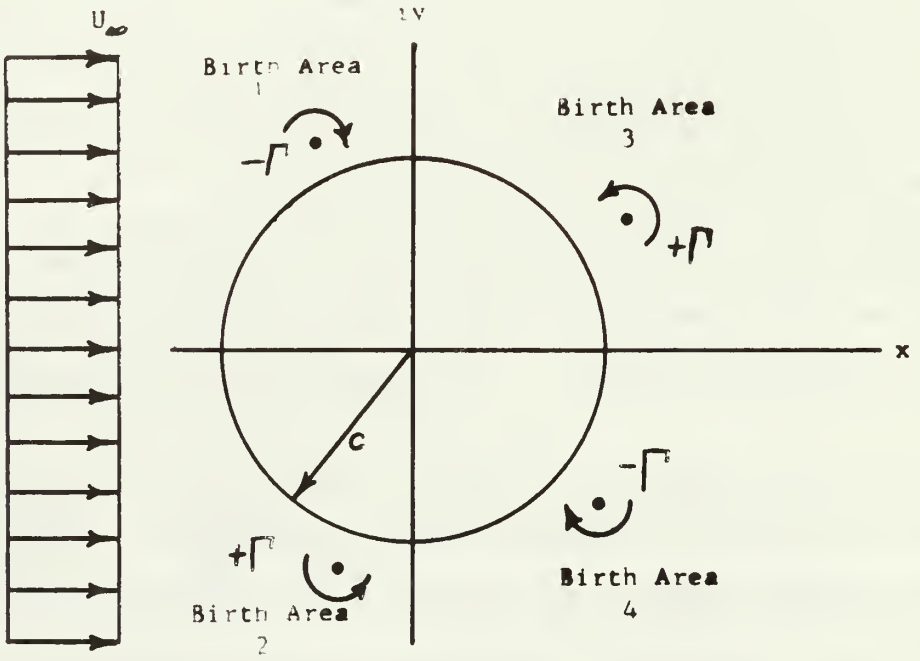

Figure 14 Configuration of Model B Showing the Birth Areas and the Rotation Direction of the Vortices Born Therein.

for about three percent of the drag. The two terms being discussed are equivalent to the terms inside the first and second sets of brackets on the right hand side of equation (47). Since in this case M = N, the rest of the terms become applicable to this model. Therefore,

$$C_D + iC_L = 2\pi \dot{U}_\infty + i \sum_{n=1}^{N} \Gamma_n \left[u_n + iV_n - (u_{in} + iV_{in}) \right] . \quad (49)$$

The system of basic equations is now complete, and like Model A, it has been programmed to allow asymmetry. The programming of equation (49) does not include the $\dot{U}_\infty$ term since during the study conducted for this thesis, only a uniform, steady cross-flow of $U_\infty = 1$ was considered. It is simple enough to include this term, but then the deleted terms may become more important such that some kind of determination might have to be made for them. It would be hoped that by the time these terms would be needed, the uniform flow problem would be solved and its results would be used to make this determination.

This program is also written in FORTRAN IV, but unlike that of Model A, its calculation is carried out in only seven digits. The program, as written, will allow as many as one hundred nascent vortices to be born in any of the four birth areas during any given run. A run, which goes through the complete 100 time steps allowed, requires approximately 30 minutes of computer execution time. Since the propagation aspects of the model are handled by Eulerian approximations, this could have serious error implications. However, since an approximate solution is all that is presently desired, it is felt that these errors are acceptable. The large execution time is due primarily to the large number of vortices involved in the evaluation of (29), particularly at high $\frac{U_\infty t}{c}$.

This must be done for each vortex, during each time step. The model also has the option of considering only the primary birth areas, 1 and 2, rather than all four.

Before discussing some of the output from this model, the general method of determining vortex strengths must be considered. It has been shown by Fage and Johansen [42] that in the absence of back-flow and in a steady flow situation, the amount of vorticity leaving a separation zone can be accurately approximated by

$$\left| \frac{\partial \Gamma}{\partial t} \right| = \left| 0.5 V^2 \right| \quad , \tag{50}$$

where V is the fluid velocity just outside the boundary layer at the point of separation. Thus, making the time steps, Δt, small enough, the nascent vortex strength can be given by

$$\left| \Gamma_n \right| = \left| 0.5 V^2 \Delta t \right| \quad . \tag{51}$$

The key problem now is to decide where and how to measure V and where the nascent vortex of this strength is placed. The four birth area model purports to take care of back-flow through the use of the secondary birth areas, 3 and 4. Now since the maximum velocities on or near the cylinder's surface in these regions are very small as compared to the velocities in the primary regions, the effect of making some assumptions here is not too critical. The assumption made in most of the computer runs was that the maximum back-flow velocity on the cylinder's surface in that birth area was used in equation (51). From experimental studies, these back-flow vortices appear to separate at points located

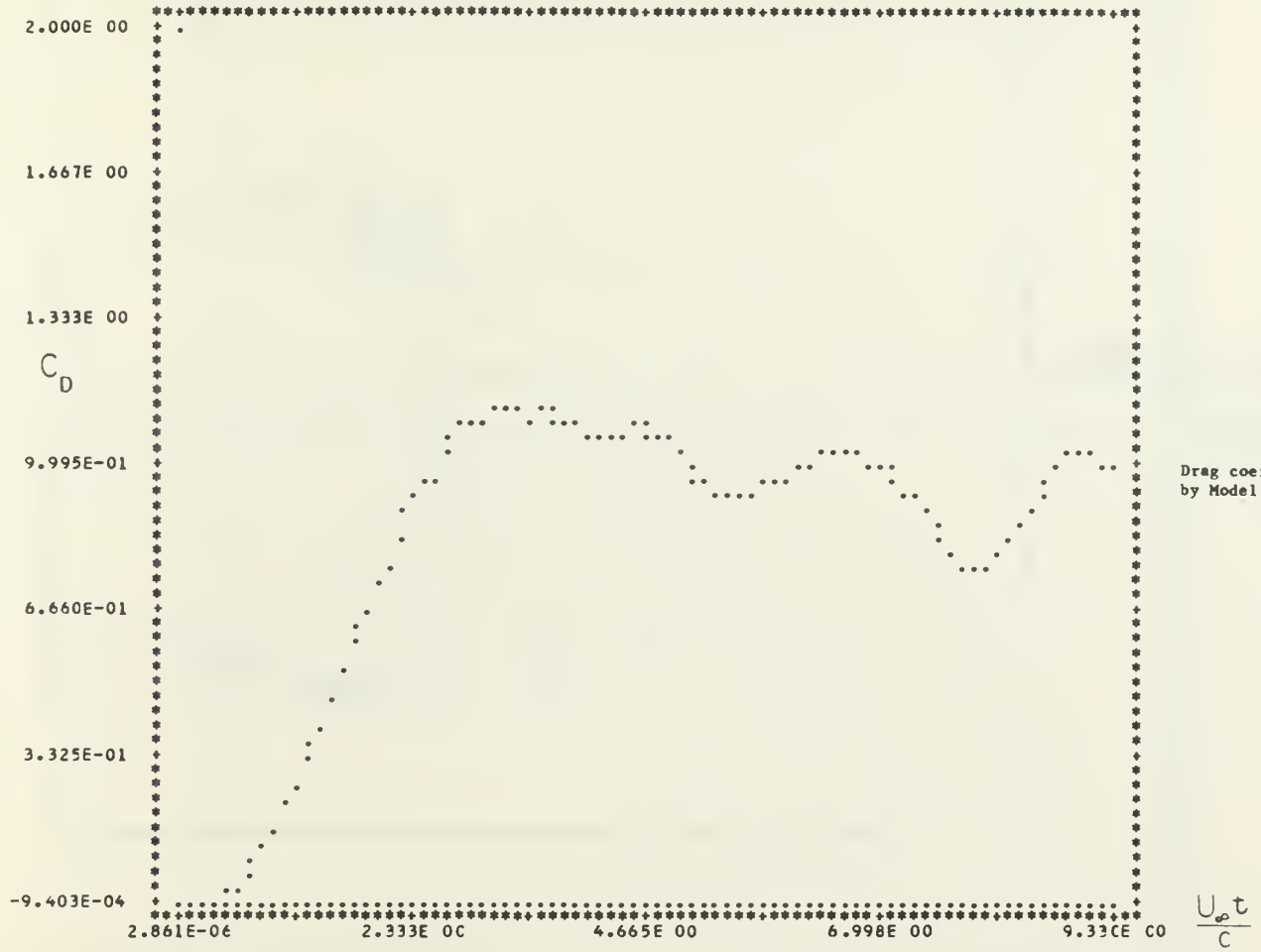

Figure 15

Drag coefficient curve generated by Model B for an asymmetric case.

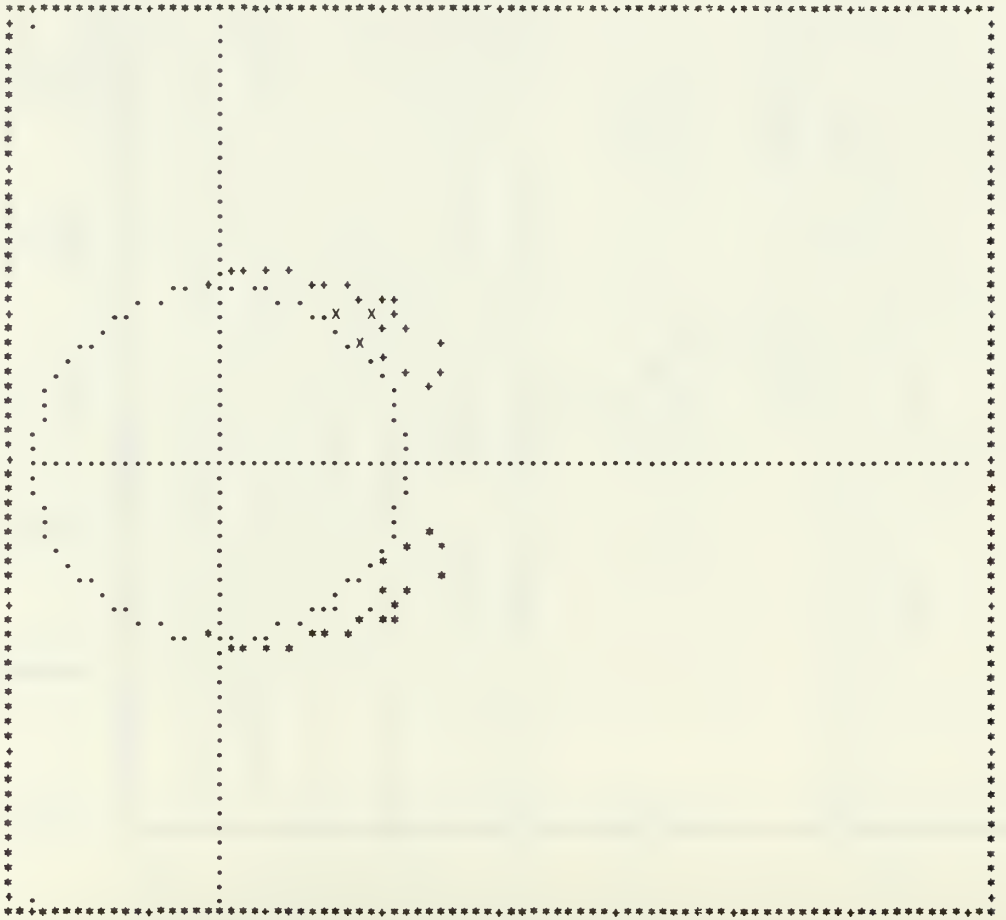

Figure 16

Wake development corresponding to $\frac{U_\infty t}{c}$ = 1.8 in Figure 15.

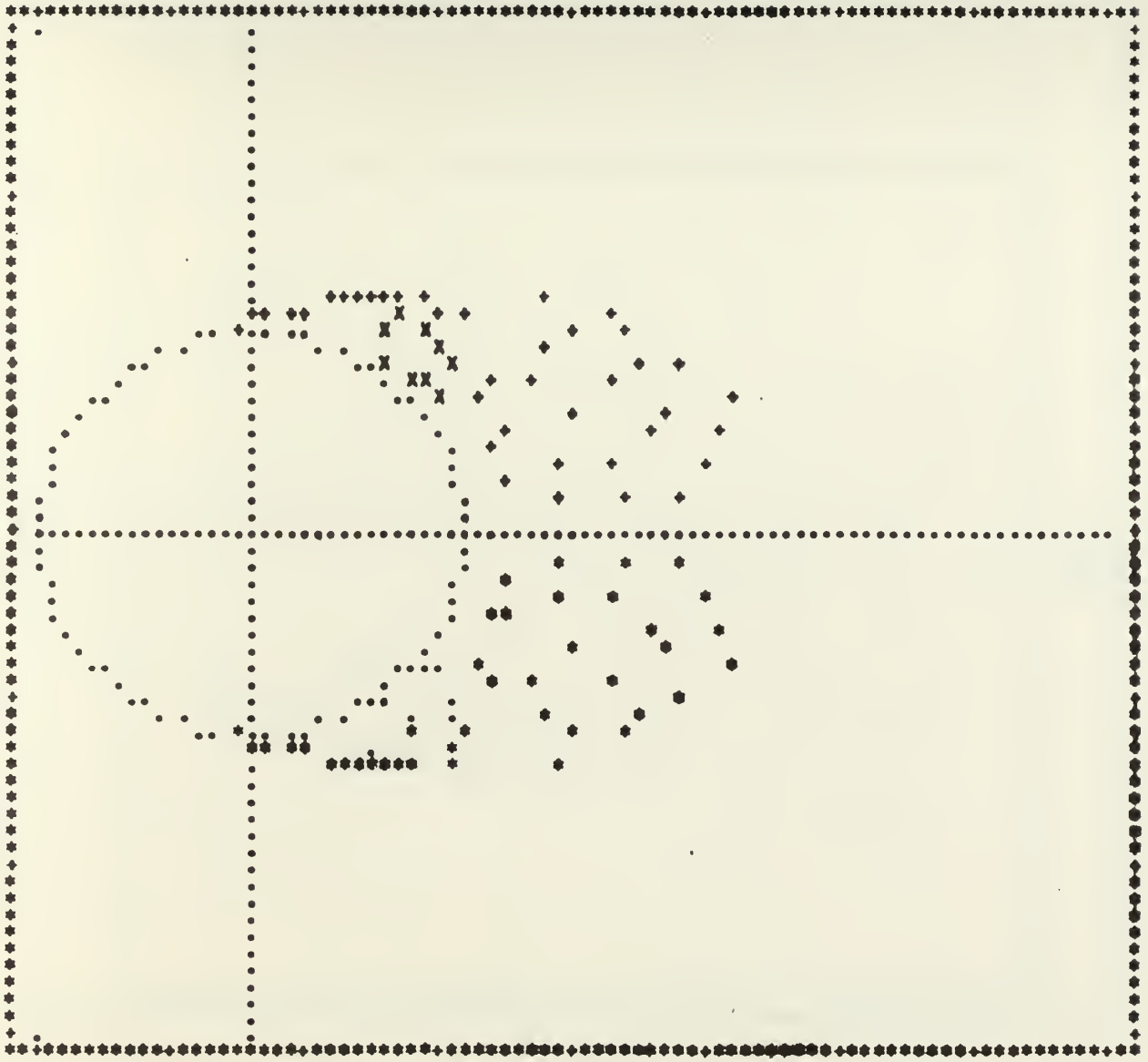

Figure 17

Wake development corresponding to $\frac{U_\infty t}{c} = 4.6$ in Figure 15.

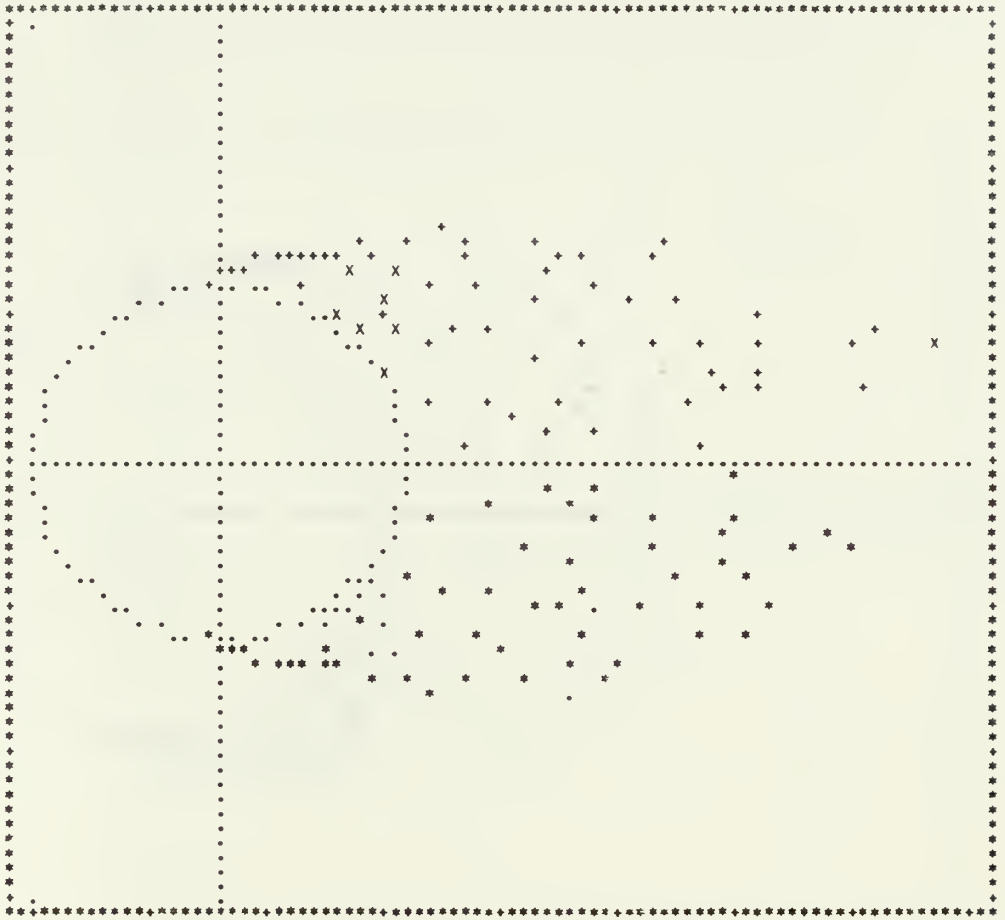

Figure 18

Wake development corresponding to $\frac{U_\infty t}{c}$ = 7.4 in Figure 15.

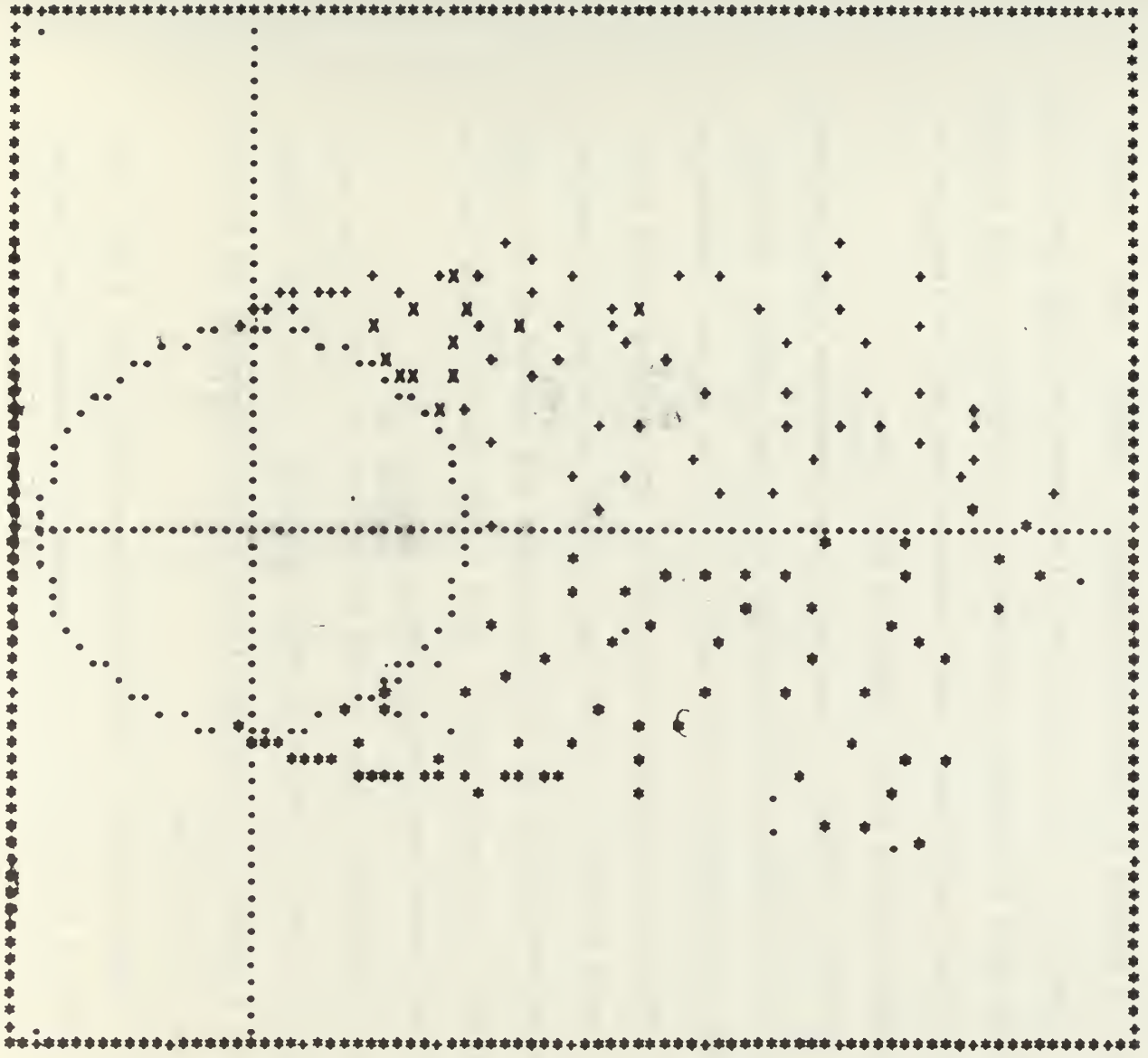

Figure 19

Wake development corresponding to $\frac{U_\infty t}{c} = 9.3$ in Figure 15.

about $\pm$ 55 degrees from the positive x-axis on the cylinder's surface; hence, these are the assumed angles for the model. Instead of placing them on the surface, however, the model was generally programmed to place them at about 0.01c. This still leaves the problem of placing the primary nascent vortices; to the author's knowledge, no truely satisfactory method does exist. The drag curves, and flow patterns shown in the figures of this subsection were all generated using arbitrarily selected placement locations. The maximum velocity found on cylinder's surface in these birth areas was, however, used to compute the strength from (51). For all the birth areas, the sign given to the resultant value of strength is as depicted in Figure 14. This sign convention comes from boundary layer considerations and the direction of the maximum velocity in the respective birth area.

In order to prevent the extremely high convective velocities that can occur if and when two vortices get too close together, the model provides for coalescence and cancellation of the vortices depending on their comparative strengths. The vortices are combined if the algebraic sum is not zero, in which case they are cancelled, by creating a new vortex having that sum for its strength and located at the "center of gravity" of the two vortices. With a great many vortices this procedure can consume much computer execution time.

As in the case of Model A, any future user of this program would of necessity have to have an intimate knowledge of its inner workings. Appendix A contains a brief description of the parameters and the major sections of the program.

This model was used to test a great many ideas using a great many runs. Figures 15 through 19 are presented to illustrate the kind and

type of output the model will generate. The program included with this thesis generated these figures. The primary vortices are injected into the stream at points whose angles from the positive x-axis are $\pm$ 95 degrees, and are located 0.045c from the cylinder's surface.

IV. DISCUSSION OF RESULTS AND CONCLUSIONS

The primary result of this study is that reasonably representative drag coefficient curves can be generated using the two mathematical models developed. This is particularly true in the case of Model B, where the curves can be generated for both symmetric and asymmetric wake development. Figures 15 through 19 show the drag curve and wake development for an asymmetric wake case, while Figures 20 through 24 show the symmetric wake case. The assumptions involved only the placement locations of the nascent vortices and the coalescence distance used to prevent abnormally high velocities if two nascent vortices come too close together. The symmetric wake case was generated by making the same assumptions for placement location as in the asymmetric case, however, the coalescence distance was increased. Where this distance was 0.06c for asymmetry, in the symmetric case it was 0.08c. These values were arbitrarily selected.

In the case of Model B, the curve in Figure 15 was generated by assuming asymmetric generation of vorticity and assuming the vortex sheet is cut, freeing the core, when $\left|\dot{\Gamma}_n\right|$ reached its first post-peak minimum. The amount of asymmetry and the time of sheet cutting is loosely based on the results of Sarpkaya's experimental work [24].

Aside from the above results, numerous computer runs were made in an effort to develop a method by which the separation characteristics could be analytically described. Though these attempts were not entirely successful, some information was gained about the character of the vortex sheet origin and the requirements necessary to produce a reasonably realistic drag curve and wake structure. Of the two models,

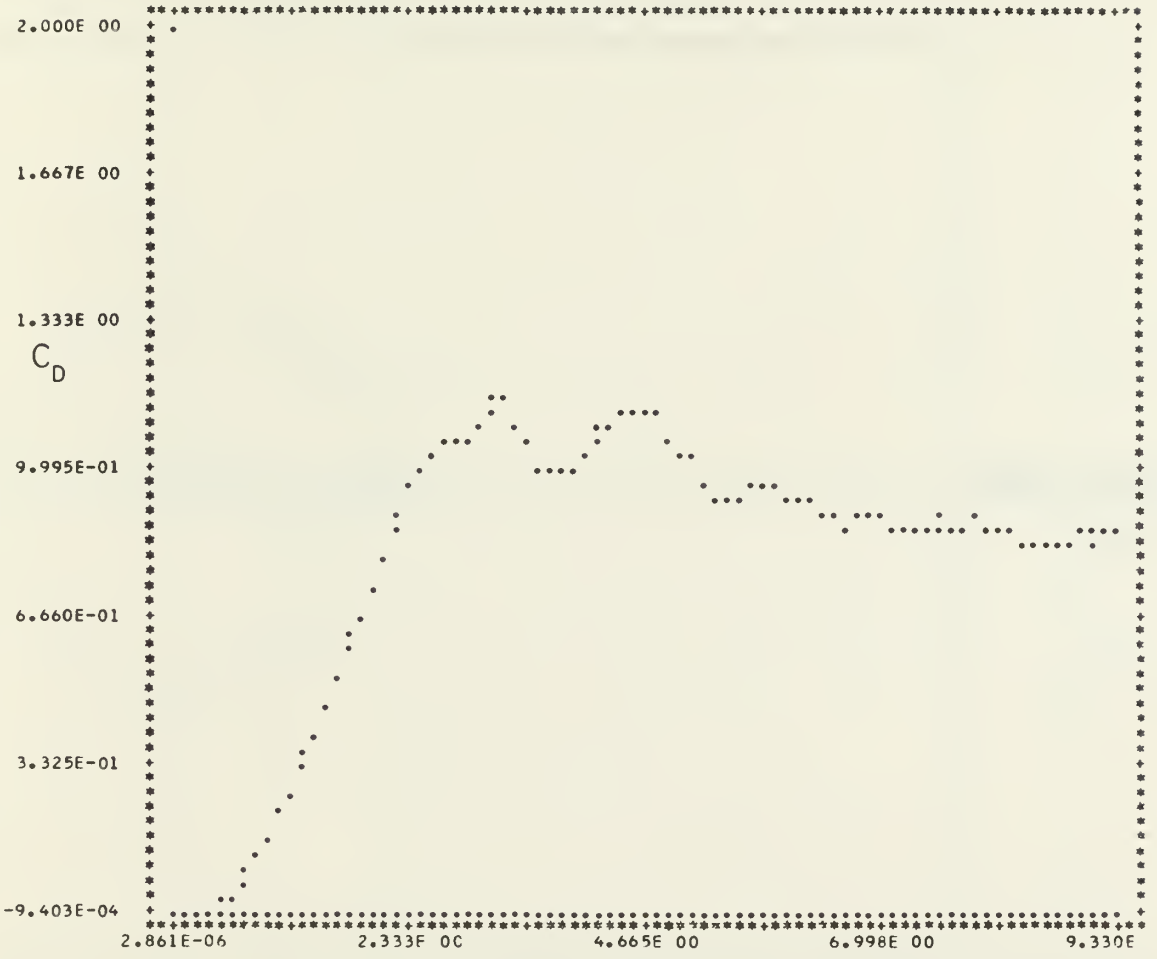

Figure 20

Drag coefficient curve generated by Model B for a symmetric case.

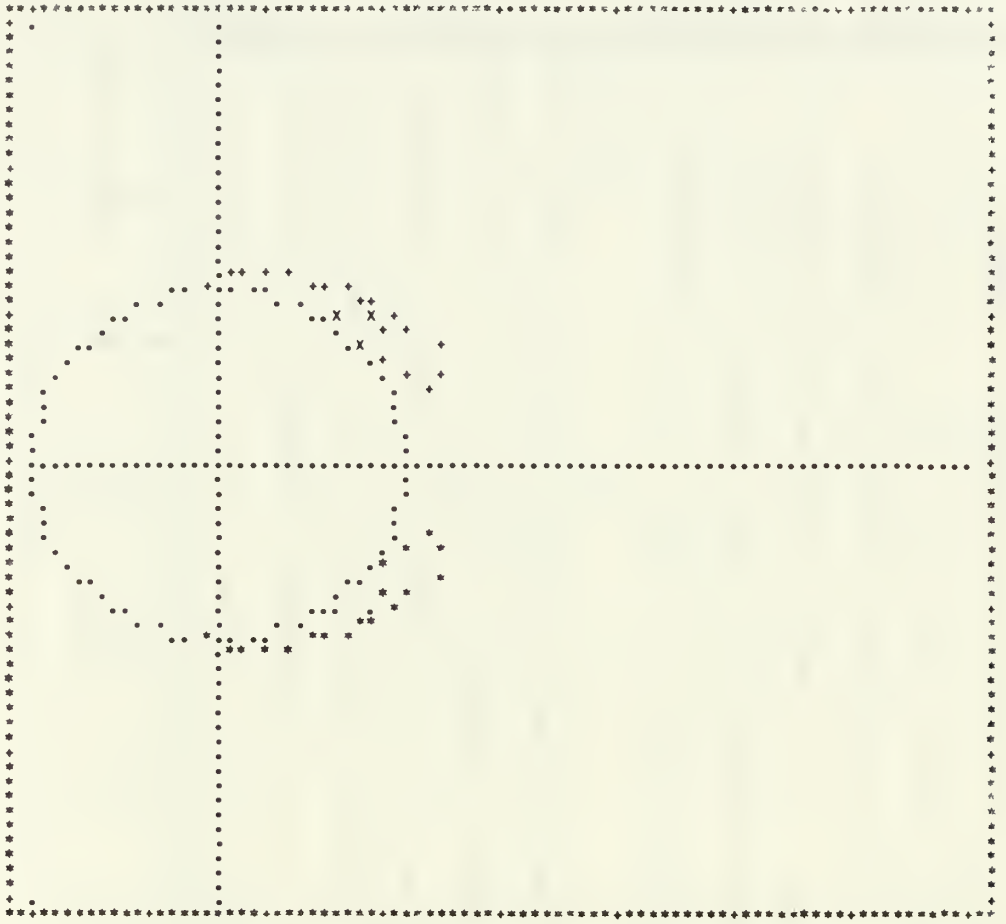

Figure 21

Wake development corresponding to $\frac{U_\infty t}{c}$ = 1.8 in Figure 20.

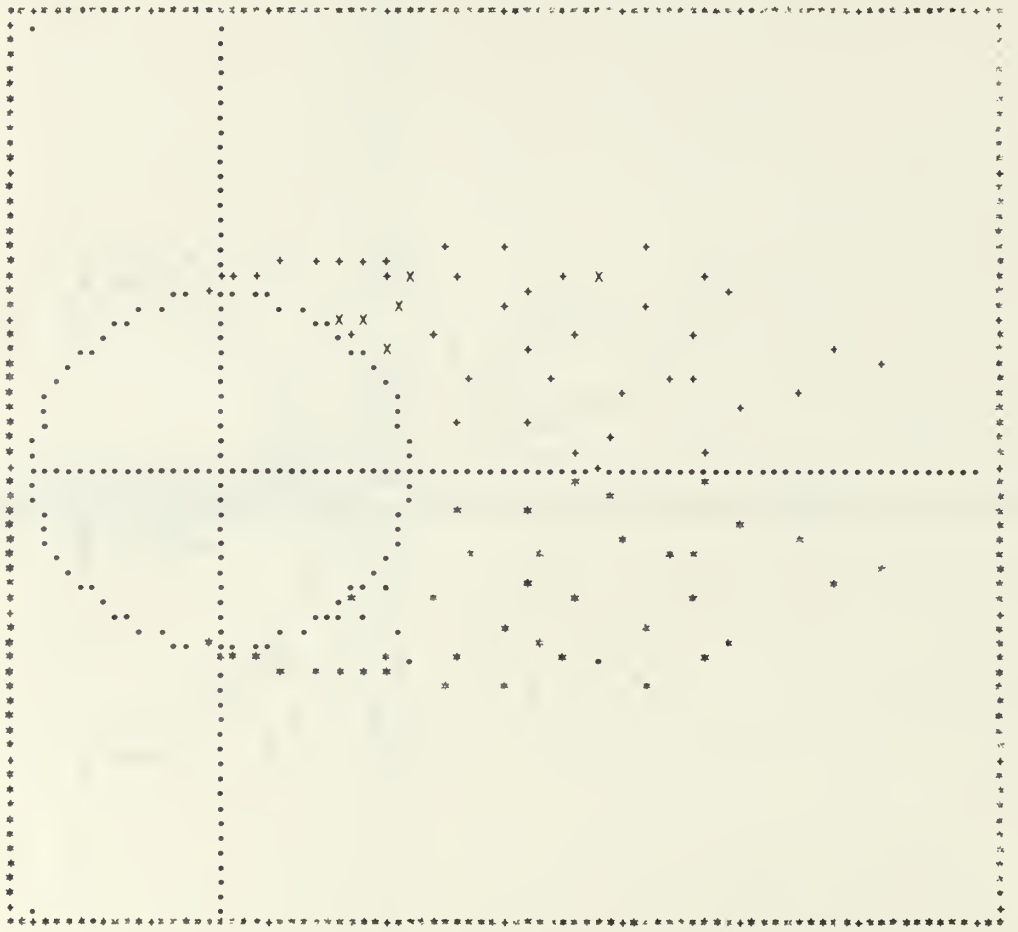

Figure 23

Wake development corresponding to $\frac{U_\infty t}{c} = 7.4$ in Figure 20.

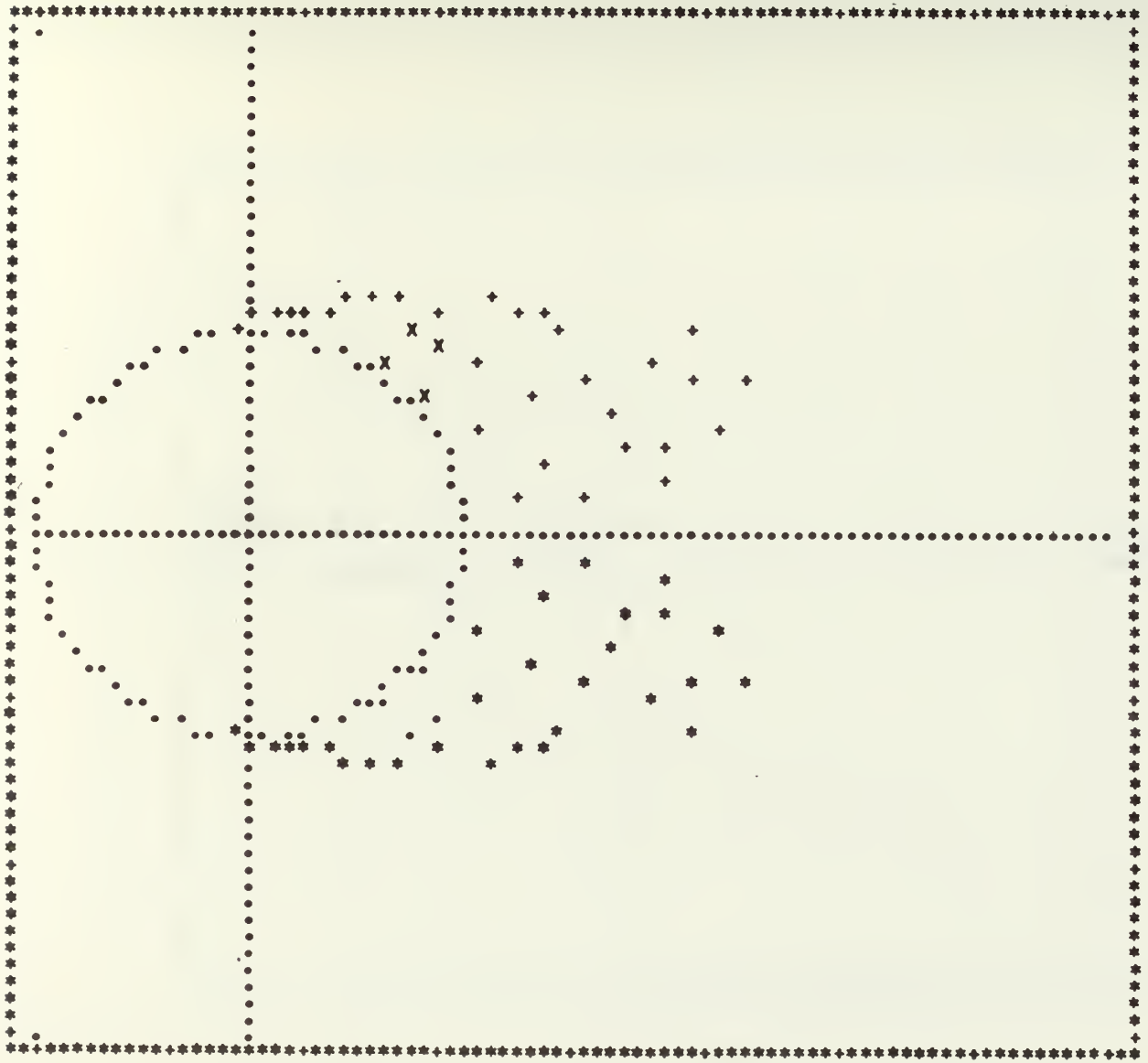

Figure 22

Wake development corresponding to $\frac{U_\infty t}{c} = 4.6$ in Figure 20.

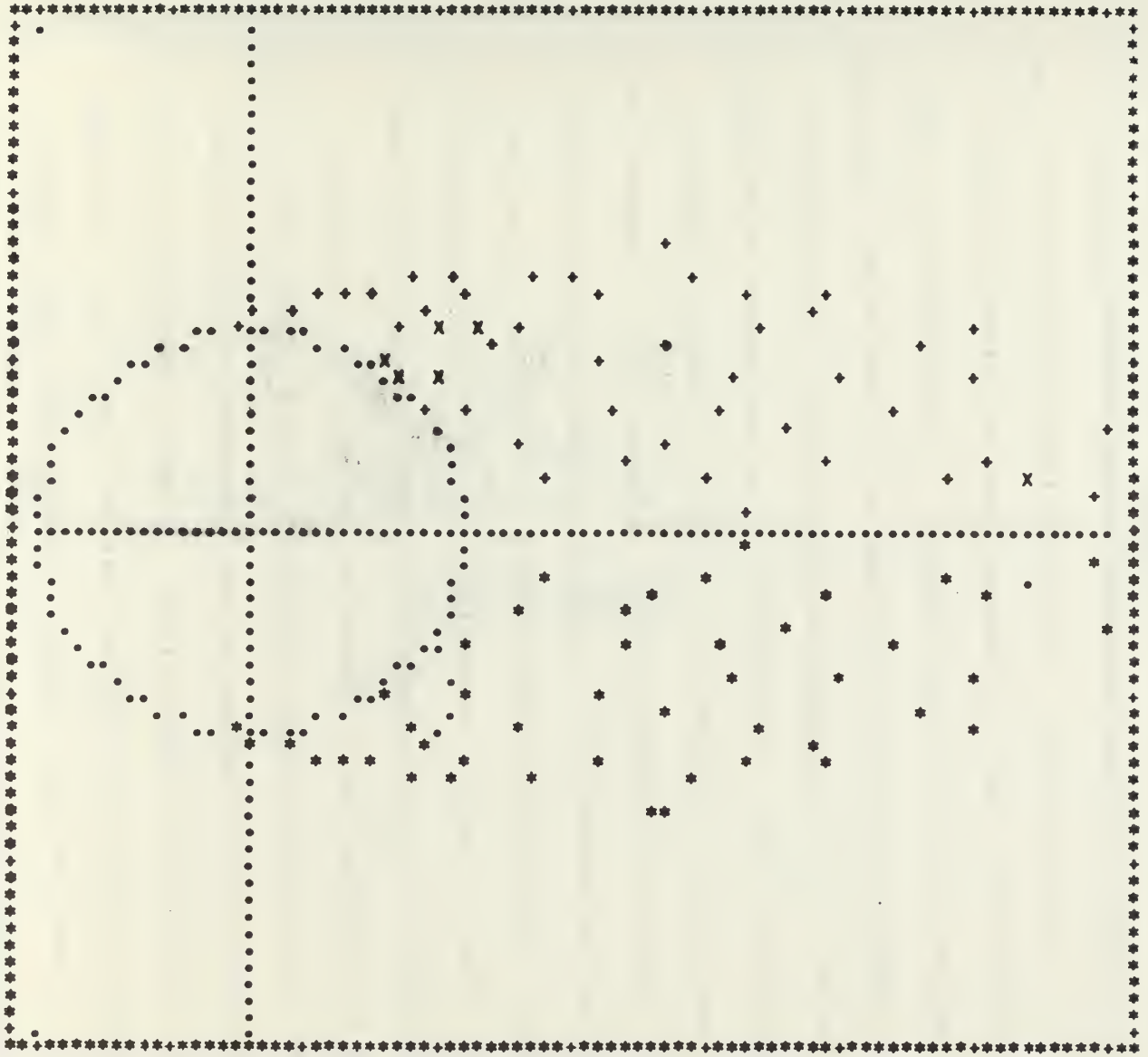

Figure 24

Wake development corresponding to $\frac{U_\infty t}{c} = 9.3$ in Figure 20.

Model B was the most illuminating since the concept of a string of small strength vortices is more nearly like nature than is the concept upon which Model A is based. As a result, Model B was utilized more extensively in this study. It is for this reason then, that the majority of the remaining remarks in this section will be concerned with results from this model.

Of the various factors that go into the final description of the wake and computation of a drag curve, it was found that the location at which the nascent vortices are placed in Model B is by far the most critical parameter involved. Of the two items which determine the location of the vortex, angle and distance from the cylinder's surface, it appears that the latter is more important. It was observed that there was a tendency for the primary vortices to "hug" the surface (Figure 25), all the way to the rear stagnation point, if their initial placement distance was too small. "Too small" in this case appears to mean less than about 0.03c. In Figure 25, the nascent vortices were placed at 0.01c away from the cylinder's surface, at angles of $\pm$ 95 degrees. Increasing or decreasing $|\Gamma_n|$ did not seem to have much effect on this tendency. Changing the secondary vortex birth location, for a fixed primary location, did not prevent this "hugging," nor did it appreciably change the roll-up tendency of the primary vortices if they were being shed reasonably well. Though the resulting wake formation in Figure 25 is not that desired for Reynolds numbers less than about 2×10^5, there is a striking resemblance to real flow patterns evident in the super critical Reynolds number region. By making approximately 50 computer runs in which only the birth location of the primary vortices was varied, it was found that a distance of about 0.045c and an angle of about $\pm$ 95 degrees from the rear stagnation point, produced the most

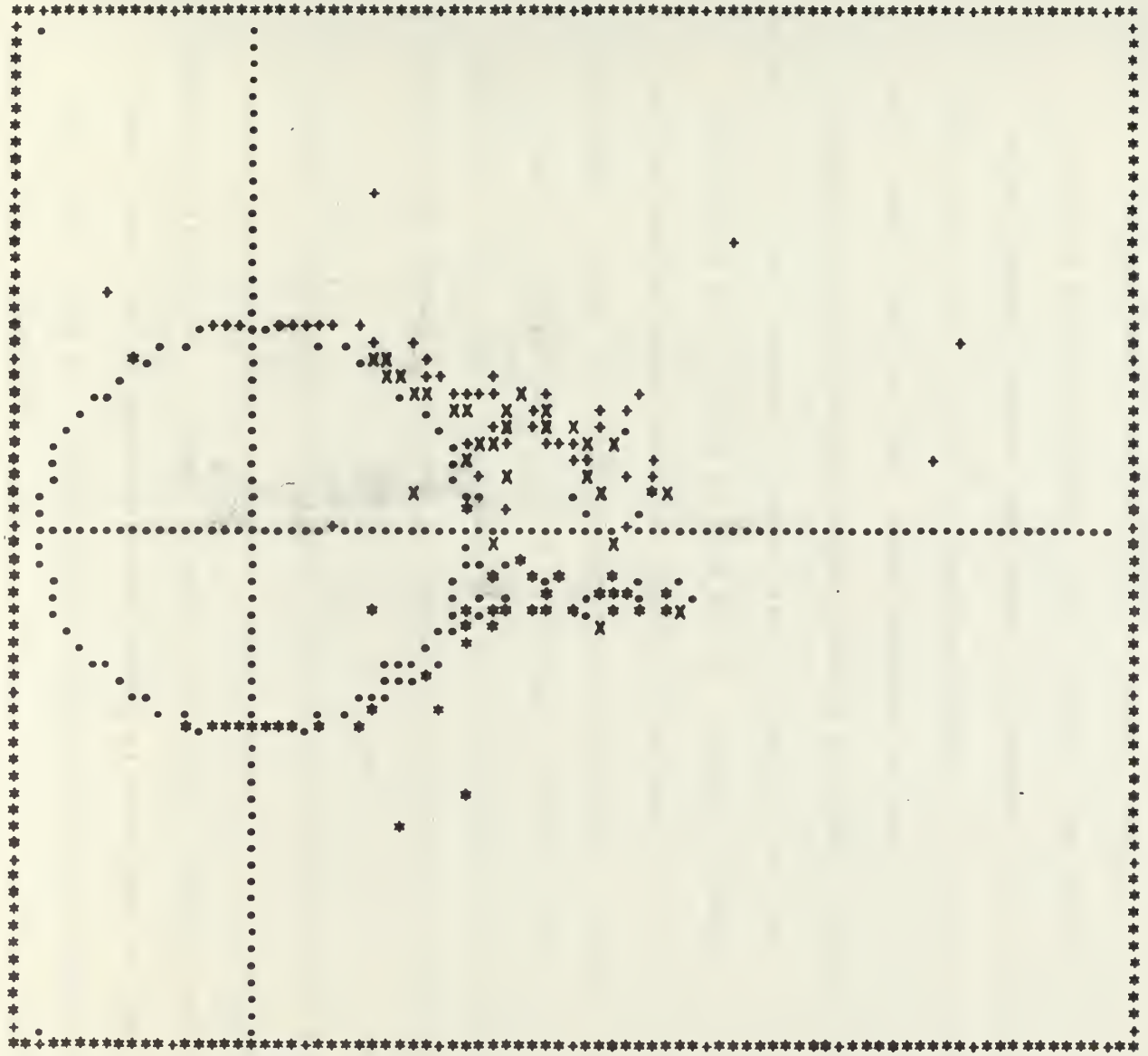

Figure 25

Wake development showing "hugging" due to vortex placement distance being too close to cylinder's surface.

reasonable flow patterns and drag curve; Figures 15 through 19. This shedding angle is comparable with the observed angle of separation of about 98 degrees (measured in the same manner). The distance from the cylinder's surface however, is about twice the value predicted by the Blasius series solution of the boundary layer equations. It is suggested, therefore, that any future work in this area might well begin with a comprehensive study of the boundary layer transition region. A consideration of these regions above flat plates could be the starting point since the added problem of body curvature is not present.

It does not seem possible, after studying the works of many investigators, that Thompson [13] could have produced asymmetry with equations that are in themselves symmetric, were it not for the very real problem of computer round-off error. The fact that Figure 15 exhibits asymmetry would be of deep concern had Abernathy and Kronauer [10] not found that strings of small strength vortices would in fact roll-up in an asymmetric manner. In the case of Model B, the predominate errors are due to assuming Eulerian propagation approximations and not from round-off. The Eulerian approximations are, of course, symmetric, so it is assumed that the asymmetry is in fact due to the interactions Abernathy and Kronauer observed. Interestingly enough, when using Model B asymmetry was not always produced, which leads to another conclusion; there is a critical combination of separation points and spacing distance between sheets to cause the necessary interaction that forms the Kármán street. On the other hand, reasonable drag coefficient curves could be generated without any asymmetry present. The overall conclusion is that the drag curve is predominatly influenced by the rate of vorticity creation and its subsequent diffusion into the wake. This then, points right back to the over-riding problem of boundary layer separation.

Since Model A feeds large vortex cores rather than creating vortex sheets, it would appear this model is better suited for describing the flow at Reynolds numbers less than about 50. By increasing the angles of the sheet origin, the drag curve peak can be increased to almost any desired value. In order to make the model applicable to the higher Reynolds number range, it will be necessary to find some scientifically based method of shedding these cores. The stability criterion of von Kármán and others may provide the answers.

APPENDIX A

A. DESCRIPTION OF COMPUTER PROGRAM FOR MODEL A

The complete program for this model includes a primary or main program with seven subroutines that accompany it. There is one other subroutine that is called that is not part of the listing included with this thesis. The subroutine is PLOTP and is called from the resident subroutine library of the Naval Postgraduate School Computing Center. The subroutine is used to produce printer plots, such as Figures 15 through 19, instead of using the CALCOMP continuous line plotters as output, so it is not critical to the actual program calculations. The curve plotting subroutines or procedures of other systems could be easily substituted for this one.

At the beginning of the main program and each subroutine is a short series of comment cards that indicate the purpose of the program and generally the parameters used therein. In general, the same notation, in FORTRAN format, is used in the program as was used to develop the equations in Section III(c); for example K_1 in equation (43) is coded as K1, or $\bar{Z}_1$ is coded as Z1B. Each computational section of the main program is preceded by comment cards indicating what computations that section is performing.

Since there is no on-line function for extracting the imaginary part of a double precision complex number in the IBM version of FORTRAN IV, EQUIVALENCE statements had to be used to recover these values.

It is assumed that the reader already has a working knowledge of FORTRAN coding techniques so that a definition list of the various parameter names used in the program will be sufficient guide for future use.

The need for understanding the program is brought about by the fact that in order to study vortex sheet cutting procedures, the logic of the coding will have to be changed each time a different procedure is used. The section of this thesis entitled Computer Output, contains several pages of output that are to be used as trial run checks for a new program deck punched from the program listing.

The type specification and precision of the following parameters is clearly indicated in the first few statements of the program, so there is no need for repeating it here. All arrays are stored in vector form, i.e., the values of the array elements are stored one column following another, going from left to right in the array.

If a parameter is not found in the following list (equation parameters described above are not included) but found in the program, then it is a dummy parameter used for intermediate operations only and therefore has no special significance as compared to those listed.

B. DEFINITION OF PARAMETERS

AA(144)	Vector containing the partitioned matrix elements.
AC(12)	Vector containing the partitioned elements of the right hand side of equation (45). First six elements contain the real parts, the second six the imaginary parts.
AG(12)	Vector containing the partitioned elements of the unknowns in equation (45).
CDCL(400)	Vector containing the lift and drag coefficient values for each time step.
CV(20)	Vector containing the free vortex velocities; corresponds to ZF(20) and GF(20).
DG1(401)	Contains $\dot{\Gamma}_1$ for each time step.
DG2(401)	Contains $\dot{\Gamma}_2$ for each time step.

DG1MAX DG2MAX DG1MIN DG2MIN	Contains the values of the maximum and minimum $\dot{\Gamma}_1$ and $\dot{\Gamma}_2$ for a run; used to determine when to cut the vortex sheets.
DT	Contains value of Δt; time step size.
DUDT	$\dot{U}$
EG	A tolerance value, usually 1.0×10^{-12}.
EGS	EG squared.
GF(20)	Vector containing the values of the strengths (Γ_m) of the free vortex cores; corresponds to the position vector ZF(20).
GG1	Value of Γ_1 during a time step.
GG2	Value of Γ_2 during a time step.
IT	Counter value for the pass number through the program. $1 \leq IT \leq NIT$
INNB	If bottom vortex is shed during a time step, this parameter equals 0, if vortex is not shed, it equals 1. It is used to determine whether a new vortex should be started from bottom sheet.
INNT	Has same indication as INNB, but applies to the top vortex and sheet.
NCHDT	Indicates the pass at which the value of DT is changed to prevent instabilities in the solution.
NFV	Counter used to indicate the number of free vortices existing. $NFV \leq 20$
NFVT	Contains the maximum value of NFV during any run. If $NFVT > 20$, the DIMENSION statement must be changed as indicated in the program.
NIT	Contains the maximum value of IT during any run. If $NIT > 400$, the DIMENSION statement must be changed as indicated in the program.
NKER(2)	Vector containing the matrix singularity determination from subroutine INVERT. NKER(1) applies to equation (42) while NKER(2) applies to equation (45).
NPL	Indicates number of passes through program between wake development plots.

NPRT	Indicates number of passes through program between status printouts.
PI	π
PI2	π^2
T(401)	Contains value of total elapsed $\frac{U_\infty t}{c}$ for each time step.
ZA(36)	Vector containing the values of the elements in the coefficient matrix of equation (45).
ZAA(4)	Same as ZA(36), but applies to equation (42).
ZC(6)	Vector containing the elements of the column matrix on the right hand side of equation (45).
ZCC(2)	Same as ZC(6), but applies to equation (42).
ZCP(100)	Contains the circle circumference points and scaling points used for plotting purposes.
ZF(20)	Contains the free vortex positions during a time step; corresponds to the vector GF(20) on a one for one basis.
ZG(6)	Contains the values of the unknowns in equation (45).
ZGG(2)	Same as ZG(6), but applies to equation (42).

APPENDIX B

A. DESCRIPTION OF COMPUTER PROGRAM FOR MODEL B

This program is much shorter from a coding standpoint than is Model A. There is only one subroutine besides the plotting routine, PLOTP, needed for operation of the program, and it is included in the listing. The general comments previously made for Model A, apply equally well here. The only major difference, besides the concepts upon which the two are based, is that this program is written using single precision arithmetic rather than double precision as in the case of Model A.

B. DEFINITION OF PARAMETERS

AVN(4) — Values, in degrees, of the nascent vortex placement angles actually used in the calculations. AVN(1) applies to birth area 1; AVN(2) applies to birth area 2, etc.

CDCL(100) — Same as Model A.

CLOCKR, CLOCKS, CLOCKT — These parameters are used to determine the actual elapsed computer time for each pass and run; can be eliminated from the program without hurting its purpose.

CVV(400) — Contains the value velocities for each vortex during a time step.
CVV(1) to CVV(100) Birth Area 1
CVV(101) to CVV(200) Birth Area 2
CVV(201) to CVV(300) Birth Area 3
CVV(301) to CVV(400) Birth Area 4

DTP — Same as DT in Model A

EG — Same as Model A

EGS — Same as Model A

GK(400) — Contains the value of the vortex strengths during a time step; corresponds to CVV(400) on a one for one basis.

IT — Same as Model A

ITIME(0)	An IBM on-line function used to make real time available to a FORTRAN IV program; can be eliminated without hurting the purpose of the program.
MPA	The value of nascent vortex placement distance as a function of RE as generated by formulas from Sarpkaya's paper [35]. Used for comparative purposes.
MP(4)	Contains the actual nascent vortex placement distance from the cylinder's surface used in the calculations. MP(1) corresponds to AVN(1) or NVM(1); MP(2) corresponds to AVN(2) or NVM(2); etc.
MS	Radius of the arc along which the search for maximum velocities is made; usually equal to 1.0.
MZD	Minimum distance two vortices can be from each other without being coalesced or cancelled.
NB	Total number of birth areas; usually equal to 4, but if only the primary nascent vortices are being studied, then NB=2.
NCAN	Counter indicating number of vortices cancelled during a run.
NCL	Counter indicating number of vortices coalesced during a run.
NIT	Same as Model A.
NP1	Indicates number of passes, times ten, between wake development plots.
NV	Counter indicating number of primary vortices generated in birth area 1 during a run. It is also equal to the number generated in birth area 2.
NVM(4)	Contains the values, in degrees, of the maximum velocity locations resulting from a search for same in the birth areas. NVM(1) corresponds to AVN(1); etc.
PI	π
RE	Reynolds number; used in conjunction with MPA.
TP(101)	Same as T(401) in Model A.
VELM(4)	Contains the values of maximum velocity found during the search for same in the birth areas. VELM(1) corresponds to NVM(1); etc.
ZCC(100)	Same as ZCP(100) in Model A.

ZCFP(51)	Contains the cylinder's circumferential locations from 135 to 85 degrees (measured from the rear stagnation point).
ZCRP(61)	Contains the cylinder's circumferential locations from 0 to 60 degrees (measured from the rear stagnation point).
ZF(400)	Contains the locations of the vortices during a time step. Corresponds to CVV(400) and GK(400).

```
&INPUT
THETAT=  50.000000000000000    ,THETAB= -50.000000000000000    ,PCTADT= 0.49999997E-01,PCTADB= 0.49999997E-01
&END
```

IT = 1 TIME = 0.0 DELTA TIME = 0.02000 CDCL = 0.045 -0.001

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.692	0.757	0.0425	0.0525	-0.0196	0.0634	-0.482	0.607	50.00
LOWER FEEDING SHEET	0.692	-0.757	0.0360	-0.0147	0.0331	-0.0152	0.459	-0.026	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.0 , BOTTOM = 0.0

```
&OUTPUT
ZGG=      (-0.45889371487403560,0.0                    ),      (0.45889371487403560,0.0                    ),ZG=
     (0.60746564613474680,0.0                    ), (-0.26321491535613410-01,0.0                    ),
  (0.42486189511857920-01,-0.52481468219046750-01), (0.42486149511857940-01,-0.52481468219046740-01),
 (0.35972376313612490-01,-0.14736202744142230-01), (0.35972376313612520-01,0.14736202744142220-01),K11=
  (0.0                   ,0.0                    ),K12=  (0.0                   ,0.0                    ),NKER=          1,
          1
&END
```

IT = 2 TIME = 0.020 DELTA TIME = 0.02000 CDCL = 0.045 -0.001

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.693	0.758	0.0331	-0.0037	0.0610	-0.0079	-0.470	-0.262	50.00
LOWER FEEDING SHEET	0.693	-0.758	0.0322	0.0043	0.0612	0.0092	0.458	0.265	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.0 , BOTTOM = 0.0

IT = 75 TIME = 3.150 DELTA TIME = 0.05000 CDCL = 1.013 -0.086

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.986	0.746	0.1864	0.0034	0.3195	0.0018	-2.365	-0.917	50.00
LOWER FEEDING SHEET	0.971	-0.738	0.1742	-0.0006	0.3093	0.0110	2.256	0.928	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.908, BOTTOM = 0.917

```
&OUTPUT
ZGG=      (-2.252150627135655,0.0                    ),      (2.2563e6415477658,0.0                    ),ZG=
     (-0.9174945110173353,0.0                    ),      (0.4278322961557248,0.0                    ),
      (0.1463907796316207,0.9434719696712533D-02),      (0.1863907796316208,-0.9434719696712545D-02),
     (0.1742450008672087,-0.6384968473974323D-03),      (0.1742450008672087,0.6384968473974252D-03),K11=
  (0.0                  ,0.0                    ),K19=  (0.0                  ,0.0                    ),NKEP=          1,
        1
&END
```

IT = 80 TIME = 3.350 DELTA TIME = 0.05000 CDCL = 1.130 -0.101

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	1.024	0.749	0.1972	0.0127	0.3393	0.0061	-2.558	-0.954	50.00
LOWER FEEDING SHEET	1.006	-0.738	0.1835	-0.0022	0.3277	0.0089	2.442	0.968	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.945, BOTTOM = 0.958

```
&OUTPUT
ZGG=      (-2.436621219524160,0.0                    ),      (2.441e7533801347,0.0                    ),ZG=
     (-0.9540e0240450762e,0.0                    ),      (0.5684962122330597,0.0                    ),
      (0.1972169373442636,0.1265260107876291D-01),      (0.1972169373442636,-0.1265260107876291D-01),
     (0.1834998872013544,-0.2230883130392288D-02),      (0.1834998872013545,0.2230883130392237D-02),K11=
  (0.0                  ,0.0                    ),K19=  (0.0                  ,0.0                    ),NKEP=          1,
        1
&END
```

IT = 204 TIME = 9.370 DELTA TIME = 0.02000 CDCL = 1.200 -1.654

	CORE POSITION X	IY	TRUE CORE VELOCITY DX/DT	I DY/DT	DW/DZ VELOCITY U	IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.711	0.774	0.1796	0.2014	-0.0263	0.1773	0.020	-0.061	50.00
LOWER FEEDING SHEET	1.613	-0.559	0.0081	0.0340	0.0713	0.0475	8.234	0.536	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.0 , BOTTOM = 0.730

FREE VORTEX CORES.

ORIGINATING FROM UPPER VORTEX SHEET					ORIGINATING FROM LOWER VORTEX SHEET				
POSITION X	IY	VELOCITY U	IV	STRENGTH	POSITION X	IY	VELOCITY U	IV	STRENGTH
2.063	1.007	0.243	0.134	-7.727	0.0	0.0	0.0	0.0	0.0

```
&OUTPUT
ZGG=  (0.1939810213263637D-01,0.0                    ),      (8.234280293874065,0.0                    ),ZG=
 (-0.6123165116299052D-01,0.0                    ),      (0.5361513612564022,0.0                    ),
          (0.1795639725761340,0.2014478242382351),          (0.1795639725761340,-0.2014478242382348),
  (0.8104965882757429D-02,0.3398863976421362D-01), (0.8104965882757432D-02,-0.3398863976421364D-01),K11=
          (10.220571584498 97,-12.18040290419294),K19=          (4.109098633912875,4.897033059289628),NKER=          1,
        1
&END
```

IT = 208 TIME = 9.450 DELTA TIME = 0.02000 CDCL = 1.239 -1.671

	CORE POSITION X	IY	TRUE CORE VELOCITY DX/DT	I DY/DT	DW/DZ VELOCITY U	IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.728	0.793	0.2550	0.2835	0.0100	0.2058	0.016	-0.046	50.00
LOWER FEEDING SHEET	1.614	-0.557	0.0115	0.0328	0.0745	0.0464	8.277	0.537	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.0 , BOTTOM = 0.730

FREE VORTEX CORES.

ORIGINATING FROM UPPER VORTEX SHEET					ORIGINATING FROM LOWER VORTEX SHEET				
POSITION X	IY	VELOCITY U	IV	STRENGTH	POSITION X	IY	VELOCITY U	IV	STRENGTH
2.082	1.017	0.250	0.137	-7.727	0.0	0.0	0.0	0.0	0.0

```
&OUTPUT
ZGG=  (0.1524388085657003D-01,0.0                    ),      (8.277135271931246,0.0                    ),ZG=
 (-0.4613264180844301D-01,0.0                    ),      (0.5365371467693618,0.0                    ),
          (0.2549713955197110,0.2834645103167961),          (0.2543713855197109,-0.2834645103167963),
  (0.1153301888490483D-01,0.3277141627513760D-01), (0.1153301888490490D-01,-0.3277141627510375D-01),K11=
          (10.16662932550766,-12.11611702324740),K19=          (4.132724166121046,4.925188721756661),NKER=          1,
        1
&END
```

IT = [illegible] TIME = [illegible] DELTA TIME = .2000 CDCL = 1.77[illegible] 0.06

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.67[illegible]	0.761	-0.0038	0.0006	0.0004	-0.0002	-0.237	-0.044	50.00
LOWER FEEDING SHEET	0.665	-0.769	-0.0032	0.0148	0.0824	0.0017	0.018	0.077	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.057, BOTTOM = 0.0

FREE VORTEX CORES.

ORIGINATING FROM UPPER VORTEX SHEET					ORIGINATING FROM LOWER VORTEX SHEET				
POSITION X	POSITION IY	VELOCITY U	VELOCITY IV	STRENGTH	POSITION X	POSITION IY	VELOCITY U	VELOCITY IV	STRENGTH
3.610	1.801	0.459	0.246	-7.727	4.078	2.650	1.749	-0.456	-0.069
2.394	-0.066	0.270	0.181	8.550	0.948	-1.044	0.656	-0.617	-0.012

```
&OUTPUT
ZGG=      (-0.28282102107113280,0.0                  ),  (0.14883139748745950-01,0.0                  ),ZG=
 (-0.43720837150933180-01,0.0                  ),  (0.77076594798557200-01,0.0                  ),
 (-0.38554022621613770-02,0.61589305483213720-03),(-0.38554022621613760-02,-0.61589305483214570-03),
 (-0.82339742796400450-02,0.14797532228918290-01),(-0.92336742796400540-02,-0.14797532228918210-01),K11=
          (0.24216432079308360,-0.28860019330326440),K13=          (-5.3711163081208340,-6.4010471564443501),NKER=          1,
           1
&END
```

IT = 328 TIME = 13.560 DELTA TIME = 0.02000 CDCL = 1.767 0.914

	CORE POSITION X	CORE POSITION IY	TRUE CORE VELOCITY DX/DT	TRUE CORE VELOCITY I DY/DT	DW/DZ VELOCITY U	DW/DZ VELOCITY IV	CORE STRENGTH	D-GAMMA DT	FEEDING SHEET ORIGIN (DEGREES)
UPPER FEEDING SHEET	0.67[illegible]	0.761	-0.0038	0.0006	0.0004	-0.0001	-0.300	-0.040	50.00
LOWER FEEDING SHEET	0.665	-0.768	0.0049	0.0233	0.0911	0.0168	0.026	0.101	-50.00

NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORES, WHEN ABS(D-GAMMA/DT) IS: TOP = 0.057, BOTTOM = 0.0

FREE VORTEX CORES.

ORIGINATING FROM UPPER VORTEX SHEET					ORIGINATING FROM LOWER VORTEX SHEET				
POSITION X	POSITION IY	VELOCITY U	VELOCITY IV	STRENGTH	POSITION X	POSITION IY	VELOCITY U	VELOCITY IV	STRENGTH
3.647	1.821	0.462	0.248	-7.727	4.216	2.610	1.671	-0.569	-0.069
2.416	-0.051	0.268	0.177	8.550	1.003	-1.093	0.747	-0.615	-0.012

```
&OUTPUT
ZGG=      (-0.28615711436957087,0.0                  ),  (0.25739465118323510-01,0.0                  ),ZG=
 (-0.39942490270340,9400-01,0.0                  ),  (0.10074992255289920,0.0                  ),
 (-0.37790321364421430-02,0.61687006583887680-03),(-0.37790321364421430-02,-0.61687006583887700-03),
 (0.48332609217177150-02,0.23331161737130380-01),(0.48332609217177190-02,-0.23331161737130430-01),K11=
          (0.11568538402316650,-0.13785346733522660),K13=          (-5.1267098107672650,-6.1097748351698700),NKER=          1,
           1
&END
```

```
&INPUT
RE=  30625.000     ,TMAXP=  10.000000     ,NFOR=          2,NCOAL=          2,NNAXI=          1,NTNAXI=          1,NOTP=          3
&END

IT =     1          TP =    0.0          DTP =    0.09425      MPA =    0.01000    MZO =    0.06000
                    TIME STEP MULTIPLIER =  3

                                 TRUE MAX.        THETA          MAX. CROSS      NASCENT VORTEX
                                 VELOCITY       USED IN          & BACFLOW         PLACEMENT
                                 LOCATION     CALCULATIONS      VELOCITIES      DISTANCE (M)

          **FRONT,TOP    **          90            95.00           2.0000          0.045000
          **FRONT,BOTTOM**         -90           -95.00           2.0000          0.045000
          **REAR,TOP     **          0              0.0            0.0             0.0
          **REAR,BOTTOM  **          0              0.0            0.0             0.0

COCL =     0.0      0.0          ZS1 =    0.0      0.0      ZS2 =     0.0      0.0

IT =     6          TP =    0.471        DTP =    0.09425      MPA =    0.01000    MZD =    0.06000
                    TIME STEP MULTIPLIER =  3

                                 TRUE MAX.        THETA          MAX. CROSS      NASCENT VORTEX
                                 VELOCITY       USED IN          & BACFLOW         PLACEMENT
                                 LOCATION     CALCULATIONS      VELOCITIES      DISTANCE (M)

          **FRONT,TOP    **         108            95.00           1.8096          0.045000
          **FRONT,BOTTOM**        -108           -95.00           1.8096          0.045000
          **REAR,TOP     **          0              0.0            0.0             0.0
          **REAR,BOTTOM  **          0              0.0            0.0             0.0

COCL =     0.026    0.000        ZS1 =   -0.368    0.000    ZS2 =    -0.394   -0.000

                                                  VORTEX SHEETS

     1. FRONT, TOP                                2. FRONT, BOTTOM                             3. REAR, TOP                                 4. REAR, BOTTOM
 POSITION     VELOCITY    GAMMA *    POSITION     VELOCITY    GAMMA *    POSITION     VELOCITY    GAMMA *    POSITION     VELOCITY    GAMMA
 0.46   0.96  1.15 -0.62 -0.1881   0.46  -0.96  1.15  0.62  0.1881   0.0    0.0   0.0   0.0   0.0  I  0.0    0.0   0.0   0.0   0.0
 0.33   1.03  1.35 -0.40 -0.1621   0.33  -1.03  1.35  0.40  0.1621   0.0    0.0   0.0   0.0   0.0  I  0.0    0.0   0.0   0.0   0.0
 0.19   1.06  1.42 -0.21 -0.1591   0.19  -1.06  1.42  0.21  0.1591   0.0    0.0   0.0   0.0   0.0  I  0.0    0.0   0.0   0.0   0.0
 0.04   1.06  1.44 -0.01 -0.1571   0.04  -1.06  1.44  0.01  0.1571   0.0    0.0   0.0   0.0   0.0  I  0.0    0.0   0.0   0.0   0.0
-0.09   1.04  1.43  0.21 -0.1561  -0.09  -1.04  1.43 -0.21  0.1561   0.0    0.0   0.0   0.0   0.0  I  0.0    0.0   0.0   0.0   0.0

TOTAL NO. OF VORTEX PAIRS CANCELLEO =      0.TOTAL NO. CF VORTEX PAIRS COALESCENCEO =      0.

ELAPSED COMPUTING TIME FOR INTERATION =     1.01 SECONOS.  TOTAL ELAPSED COMPUTING TIME =     0.07 MINUTES.
```

IT = 50 TP = 4.618 DTP = 0.09425 MPA = 0.01000 MTD = 0.06000

TIME STEP MULTIPLIER = 3

	TRUE MAX. VELOCITY LOCATION	THETA USED IN CALCULATIONS	MAX. CROSS & BACKFLOW VELOCITIES	NASCENT VORTEX PLACEMENT DISTANCE (M)
**FRONT,TOP **	112	55.00	1.5700	0.045000
FRONT,BOTTOM	-112	-55.00	1.5764	0.045000
**REAR,TOP **	15	55.00	0.9023	0.045000
**REAR,BOTTOM **	-15	-55.00	0.8848	0.045000

CDCL = 1.096 -0.002 ZS1 = -0.582 0.007 ZS2 = -1.678 0.009

VORTEX SHEETS

1. FRONT, TOP					2. FRONT, BOTTOM					3. REAR, TOP						4. REAR, BOTTOM				
POSITION		VELOCITY		GAMMA *	POSITION		VELOCITY		GAMMA *	POSITION		VELOCITY		GAMMA *		POSITION		VELOCITY		GAMMA
1.29	0.72	0.25	0.24	-0.071I	1.30	-0.72	0.28	-0.23	0.071I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.18	0.26	-0.46	0.43	-0.162I	1.19	-0.26	-0.45	-0.42	0.162I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.70	0.35	-0.31	-0.05	-0.159I	1.69	-0.35	-0.31	0.05	0.159I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
2.16	0.48	0.20	-0.63	-0.157I	2.16	-0.48	0.20	0.62	0.157I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.80	0.87	0.79	-0.43	-0.156I	1.80	-0.88	0.80	0.37	0.156I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.49	0.98	0.90	-0.12	-0.154I	1.50	-0.98	0.90	0.14	0.154I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.94	0.60	0.39	-0.44	-0.153I	1.94	-0.60	0.39	0.45	0.153I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.38	0.88	0.59	0.16	-0.152I	1.38	-0.88	0.60	-0.16	0.152I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.67	0.72	0.43	-0.13	-0.151I	1.68	-0.72	0.45	0.13	0.151I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.15	0.79	0.40	0.46	-0.150I	1.16	-0.78	0.43	-0.45	0.150I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.50	0.56	0.09	0.11	-0.297I	1.50	-0.56	0.10	-0.10	0.297I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.08	0.64	0.08	0.66	-0.147I	1.09	-0.64	0.10	-0.64	0.147I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.16	0.46	-0.10	0.49	-0.147I	1.16	-0.46	-0.10	-0.48	0.147I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.09	0.39	-0.32	0.79	-0.036I	1.10	-0.39	-0.31	-0.78	0.036I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.43	0.35	-0.36	0.20	-0.145I	1.43	-0.35	-0.35	-0.19	0.145I	0.60	0.98	0.03	-0.13	0.148I		0.59	-1.04	-0.06	-0.08	-0.148
1.44	0.18	-0.77	0.11	-0.144I	1.44	-0.18	-0.77	-0.10	0.144I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.97	0.21	-0.58	-0.21	-0.143I	1.98	-0.21	-0.58	0.21	0.143I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.72	0.20	-0.79	-0.06	-0.142I	1.72	-0.20	-0.78	0.07	0.142I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.89	0.47	-0.10	-0.26	-0.214I	1.89	-0.47	-0.09	0.25	0.214I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
2.14	0.34	-0.26	-0.49	-0.140I	2.14	-0.34	-0.25	0.50	0.140I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
2.25	0.64	0.60	-0.77	-0.139I	2.25	-0.64	0.60	0.77	0.139I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
2.00	0.80	0.84	-0.66	-0.274I	1.99	-0.80	0.83	0.66	0.274I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.71	1.05	-0.21	0.24	0.111I		0.73	-0.92	-0.33	-0.20	-0.111
0.98	1.06	0.36	-0.09	-0.402I	0.77	-1.02	-0.13	-0.13	0.271I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.83	1.03	-0.52	0.43	0.095I		0.0	0.0	0.0	0.0	0.0
1.74	0.97	1.06	-0.36	-0.134I	1.75	-1.04	1.27	0.38	0.266I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
1.70	1.07	1.41	-0.35	-0.133I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	1.41	-1.14	1.28	0.02	0.262I	0.90	0.94	-0.29	0.38	0.091I		0.91	-0.91	-0.23	-0.17	-0.091
1.40	1.16	1.29	-0.06	-0.260I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.85	1.12	0.38	0.08	-0.379I	1.00	-1.03	0.21	0.06	0.161I	0.94	0.86	0.07	0.53	0.101I		0.95	-0.84	0.17	-0.49	-0.101
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.96	-1.11	0.72	0.13	0.127I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.94	-1.17	1.36	0.07	0.125I	0.90	0.70	0.19	0.63	0.107I		0.89	-0.68	0.19	-0.54	-0.107
0.79	1.17	1.26	0.45	-0.125I	0.78	-1.13	0.95	0.20	0.125I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.71	1.16	0.68	0.23	-0.124I	0.71	-1.15	0.94	-0.09	0.124I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.64	1.15	0.57	0.03	-0.123I	0.63	-1.15	0.77	-0.07	0.123I	0.80	0.71	0.24	-0.05	0.180I		0.78	-0.70	0.26	0.24	-0.220
0.57	1.14	0.59	0.04	-0.123I	0.57	-1.14	0.58	-0.12	0.123I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.50	1.14	0.68	0.06	-0.122I	0.50	-1.14	0.69	-0.07	0.122I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.42	1.13	0.76	0.07	-0.121I	0.43	-1.13	0.78	-0.08	0.121I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.34	1.13	0.81	0.10	-0.120I	0.35	-1.13	0.82	-0.09	0.121I	0.0	0.0	0.0	0.0	0.0 I		0.0	0.0	0.0	0.0	0.0
0.26	1.12	0.84	0.12	-0.120I	0.26	-1.12	0.85	-0.12	0.120I	0.72	0.74	0.38	-0.53	0.083I		0.0	0.0	0.0	0.0	0.0
0.18	1.10	0.86	0.14	-0.119I	0.18	-1.10	0.87	-0.14	0.119I	0.0	0.0	0.0	0.0	0.0 I		0.55	-0.81	0.37	0.27	-0.111

IT = 75 TP = 6.974 DTP = 0.09425 MPA = 0.01000 MZD = 0.06000

TIME STEP MULTIPLIER = 3

	TRUE MAX. VELOCITY LOCATION	THETA USED IN CALCULATIONS	MAX. CROSS & BACFLOW VELOCITIES	NASCENT VORTEX PLACEMENT DISTANCE (M)
**FRONT,TOP **	114	95.00	1.5007	0.045000
FRONT,BOTTOM	-114	-95.00	1.5002	0.045000
**REAR,TOP **	17	55.00	0.7877	0.045000
**REAR,BOTTOM **	-18	-55.00	0.9742	0.045000

CDCL = 1.014 0.044 ZS1 = -0.528 -0.010 ZS2 = -1.542 -0.054

VORTEX SHEETS

1. FRONT, TOP					2. FRONT, BOTTOM					3. REAR, TOP					4. REAR, BOTTOM				
POSITION		VELOCITY		GAMMA *	POSITION		VELOCITY		GAMMA *	POSITION		VELOCITY		GAMMA *	POSITION		VELOCITY		GAMMA
2.55	0.70	0.66	-0.50	-0.0711	2.53	-0.69	0.46	0.34	0.0711	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.32	1.04	1.15	-0.31	-0.0801	2.28	-1.12	1.19	0.18	0.1621	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.49	0.85	0.35	0.18	-0.1591	1.56	-0.80	0.25	-0.01	0.1591	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.05	0.69	-0.04	0.76	-0.1571	1.02	-0.66	0.06	-0.73	0.1571	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.86	0.26	-0.65	-0.06	-0.1561	2.08	-0.39	-0.14	-0.05	0.1561	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.12	0.25	-0.64	-0.13	-0.1541	2.13	-0.27	-0.56	-0.01	0.1541	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.24	0.49	-0.11	0.50	-0.1531	1.23	-0.49	-0.17	-0.44	0.1531	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.30	0.26	-0.57	-0.16	-0.1521	2.19	-0.15	-0.94	0.03	0.1521	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.96	0.39	-0.23	-0.18	-0.1511	1.76	-0.45	-0.34	-0.12	0.1511	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.79	0.63	0.44	-0.51	-0.1501	2.28	-0.72	0.45	0.16	0.1501	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.73	0.57	-0.07	0.02	-0.2971	1.81	-0.64	0.20	0.04	0.2971	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
3.01	0.80	0.80	-0.40	-0.1471	2.78	-0.77	0.75	0.59	0.1471	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.28	0.83	0.65	-0.19	-0.1471	2.23	-0.90	0.73	0.16	0.1471	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
3.01	0.94	1.08	-0.46	-0.0361	2.99	-0.82	0.90	0.59	0.0361	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.84	0.77	0.30	-0.16	-0.1451	1.95	-0.90	0.69	0.05	0.1451	0.93	0.93	0.07	0.45	0.3591	0.0	0.0	0.0	0.0	0.0
1.66	0.99	0.67	0.01	-0.1441	2.02	-1.12	1.13	0.12	0.1441	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.12	1.06	0.23	0.40	-0.1431	1.18	-1.02	0.51	-0.38	0.1431	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.42	1.08	0.72	0.02	-0.1421	1.55	-0.92	0.62	0.05	0.1421	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.42	0.59	-0.02	0.26	-0.2141	1.42	-0.61	-0.04	-0.23	0.2141	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.21	0.84	0.24	0.50	-0.1401	1.20	-0.81	0.22	-0.46	0.1401	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.12	0.35	-0.21	0.63	-0.1391	1.09	-0.34	-0.32	-0.65	0.1391	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.32	0.18	-0.57	0.17	-0.2741	1.33	-0.22	-0.59	-0.17	0.2741	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
3.21	0.65	0.52	-0.53	-0.4021	0.93	-1.12	-0.01	-0.04	0.2711	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.77	0.07	-1.05	-0.05	-0.1341	1.78	-0.20	-0.82	-0.12	0.2661	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.67	0.22	-0.63	0.15	-0.1331	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	2.96	-0.15	-0.20	0.18	0.2621	0.0	0.0	0.0	0.0	0.0 I	0.57	-0.99	0.01	0.08	-0.091
2.82	0.17	-0.36	-0.19	-0.2601	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.59	0.45	-0.13	-0.26	-0.3791	3.07	-0.70	0.58	0.65	0.1611	3.47	0.85	0.83	-0.51	0.1011	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	2.30	-0.58	0.09	0.13	0.1271	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	2.36	-0.21	-0.67	0.25	0.1261	0.0	0.0	0.0	0.0	0.0 I	1.23	-1.24	1.15	-0.24	-0.107
2.27	0.70	0.16	-0.13	-0.1251	2.46	-0.42	-0.21	0.29	0.2481	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.50	0.64	0.28	-0.17	-0.1241	2.64	-0.59	0.27	0.37	0.1241	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.71	0.87	0.24	-0.28	-0.0601	0.0	0.0	0.0	0.0	0.0 I	0.93	0.57	0.01	0.31	0.4081	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	2.80	-0.51	0.10	0.54	0.2441	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.48	0.96	-0.12	0.62	-0.2431	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	2.54	-0.88	0.79	0.50	0.1411	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
2.61	0.79	0.78	-0.58	-0.0131	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
1.74	1.24	1.29	-0.10	-0.2391	2.45	-1.11	1.16	0.53	0.2391	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	0.0	0.0	0.0	0.0	0.0 I	1.74	-0.94	0.71	0.24	-0.111

IT = 100 TP = 9.330 DTP = 0.09425 MPA = 0.01000 MZD = 0.06000

TIME STEP MULTIPLIER = 3

	TRUE MAX. VELOCITY LOCATION	THETA USED IN CALCULATIONS	MAX. CROSS & BACKFLOW VELOCITIES	NASCENT VORTEX PLACEMENT DISTANCE (M)
**FRONT,TOP **	115	95.00	1.4571	0.045000
FRONT,BOTTOM	-115	-95.00	1.4365	0.045000
**REAR,TOP **	23	55.00	0.6917	0.045000
**REAR,BOTTOM **	-23	-55.00	0.8904	0.045000

CDCL = 1.007 0.158 ZS1 = -0.488 0.093 ZS2 = -1.495 -0.075

VORTEX SHEETS

1. FRONT, TOP				2. FRONT, BOTTOM				3. REAR, TOP				4. REAR, BOTTOM			
POSITION		VELOCITY	GAMMA *	POSITION		VELOCITY	GAMMA *	POSITION		VELOCITY	GAMMA *	POSITION		VELOCITY	GAMMA
1.62	0.53	-0.09	0.34 -0.071I	2.29	-0.28	-0.71	-0.04 0.071I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
3.29	0.24	-0.75	0.13 -0.080I	3.61	-0.02	-0.49	0.26 0.162I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.52	0.90	0.55	-0.07 -0.159I	2.48	-0.87	0.67	-0.01 0.159I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
3.12	1.27	1.22	-0.53 -0.157I	3.15	-1.54	1.37	0.37 0.157I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.46	0.82	0.11	0.07 -0.156I	1.20	-0.77	0.10	-0.56 0.156I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.10	0.83	0.06	0.65 -0.154I	1.38	-0.66	-0.09	-0.33 0.154I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.67	1.29	1.17	-0.09 -0.153I	2.70	-1.50	1.33	-0.09 0.153I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.99	0.58	0.13	0.63 -0.152I	1.23	-1.08	0.44	-0.42 0.152I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.12	0.44	-0.20	0.38 -0.151I	2.12	-0.80	0.42	0.19 0.151I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.50	0.23	-0.57	0.21 -0.150I	2.34	-0.46	-0.21	0.02 0.150I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.93	0.82	0.55	-0.05 -0.297I	1.72	-0.59	-0.24	-0.10 0.297I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
4.56	0.12	0.61	-0.11 -0.147I	2.10	-0.22	-0.81	0.00 0.147I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.06	0.29	-0.51	0.03 -0.147I	3.04	-0.28	-0.23	0.15 0.147I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
4.43	0.17	0.64	0.05 -0.036I	3.36	0.06	-0.62	0.00 0.036I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.11	0.63	0.12	-0.14 -0.145I	2.60	-0.41	-0.25	0.09 0.145I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.88	0.68	0.17	-0.20 -0.144I	3.52	-0.19	-0.16	0.09 0.144I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
3.13	0.70	0.28	-0.46 -0.143I	2.79	-1.11	0.92	0.01 0.143I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.94	0.51	-0.27	-0.29 -0.142I	1.86	-0.54	-0.31	0.14 0.142I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.36	1.10	0.88	-0.02 -0.214I	2.58	-1.25	1.19	-0.28 0.214I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.75	1.11	0.91	-0.19 -0.140I	3.02	-1.29	1.32	0.30 0.140I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.77	1.42	1.39	-0.30 -0.139I	2.87	-1.50	1.43	0.27 0.139I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.98	1.25	0.92	0.06 -0.274I	1.61	-0.90	0.26	-0.07 0.274I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
3.77	0.17	0.00	-0.04 -0.402I	3.04	-1.13	0.83	0.39 0.271I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.09	1.04	0.20	0.59 -0.134I	1.50	-1.05	0.73	-0.19 0.266I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.33	0.76	-0.03	0.41 -0.133I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	1.48	-0.36	-0.41	-0.23 0.262I	0.0	0.0	0.0	0.0 0.0 I	0.91	-0.99	-0.02	-0.52 -0.280
1.15	0.02	-0.31	0.03 -0.260I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.83	0.47	-0.21	0.08 -0.379I	2.67	-0.11	-0.79	-0.07 0.161I	5.18	0.27	0.78	-0.08 0.101I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	2.19	-0.56	-0.01	-0.01 0.127I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	1.02	-0.87	0.22	-0.80 0.126I	0.0	0.0	0.0	0.0 0.0 I	3.90	-0.28	0.30	0.48 -0.107
2.53	0.65	0.28	-0.11 -0.125I	1.12	-0.50	-0.26	-0.63 0.248I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.73	0.26	-0.74	0.15 -0.124I	1.93	-0.29	-0.65	-0.07 0.124I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
2.18	1.21	1.01	-0.05 -0.060I	0.0	0.0	0.0	0.0 0.0 I	1.34	1.09	0.87	-0.19 0.408I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	1.51	-0.15	-0.81	-0.11 0.244I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.18	1.06	0.05	0.02 -0.243I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	2.51	-0.25	-0.68	-0.08 0.141I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
1.62	0.12	-0.92	0.08 -0.013I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
3.36	0.32	-0.34	-0.27 -0.239I	3.05	-0.10	-0.63	0.07 0.239I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0
0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	0.0	0.0	0.0	0.0 0.0 I	1.72	-0.47	-0.65	-0.08 -0.111

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C*************************************************************************00000010
C                                                                         00000020
C   THIS PROGRAM INTERJECTS VORTICIES (20 MAXIMUM) INTO THE WAKE USING    00000030
C   THE SINGLE FEEDING SHEET THEORY OF BRYSON (1964) WITH MODIFICATIONS   00000040
C   FOR ASYMMETRIC FEEDING AND WAKE FORMATION.                            00000050
C                                                                         00000060
C   DEFINITION OF PARAMETERS IN NAMELIST INPUT BLOCK.                     00000070
C                                                                         00000080
C     THETAT = ANGULAR POSITION OF ZT, DEGREES.                           00000090
C     THETAB = ANGULAR POSITION OF ZB, DEGREES.                           00000100
C     PCTADT = PERCENT OF CYLINDER RADIUS Z1 IS TO BE INITIALLY PLACED    00000110
C              AWAY FROM CYLINDER.                                        00000120
C     PCTADB = PERCENT OF CYLINDER RADIUS Z2 IS TO BE INITIALLY PLACED    00000130
C              AWAY FROM CYLINDER.                                        00000140
C                                                                         00000150
C*************************************************************************00000160
      IMPLICIT INTEGER(I,J,L,M,N,P,Q),REAL*8(A,B,D-H,O,R,T-Y),            00000170
     CCOMPLEX*16(C,K,Z),REAL*4(S)                                         00000180
      DIMENSION        ZC(6),ZG(6),ZAA(4),ZCC(2),CV(20),AA(144),AC(12),   00000190
     CAG(12),ZGG(2),DUM(2,6),NKER(2),DS1AA(4,4),DS2AA(12,12),AX(144),     00000200
     CDS1AX(4,4),DS2AX(12,12)                                             00000210
      COMPLEX*8 CDCL(400),ZCP(100),ZZ8                                    00000220
      REAL*4 T(401),DG1(401),DG2(401),XP(400),YP(401),                    00000230
     CPCTADT,PCTADB                                                       00000240
      REAL*8 PI,PI2                                                       00000250
      INTEGER*4 LL(144),MM(144)                                           00000260
      COMMON /ALPHA/ZF(20),GF(20),EG,EGS,NFV                              00000270
      COMMON /BETA/Z1,Z1B,Z2,Z2B,U,GG1,GG2                                00000280
      COMMON /CHARLE/ZA(36),K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,   00000290
     CK14,K15,K16,K17,K18,K19,K20,ZT,ZTB,ZB,ZBB,PI,PI2                    00000300
      EQUIVALENCE (ZC(1),ZCC(1),DUM(1,1)),(AA(1),DS1AA(1,1),DS2AA(1,1)),  00000310
     C(AX(1),DS1AX(1,1),DS2AX(1,1))                                       00000320
C                                                                         00000330
C   THE DIMENSIONS OF CV,ZF,AND GF MUST EQUAL NVFT. DIMENSIONS FOR OTHER  00000340
C   VARIABLE ARRAYS ARE:       XP(NIT), CDCL(NIT), T(NIT+1), YP(NIT+1),   00000350
C   DG1(NIT+1), DG2(NIT+1).                                               00000360
C                                                                         00000370
      NFVT= 20                                                            00000380
      NIT= 400                                                            00000390
      NAMELIST/INPUT/THETAT,THETAB,PCTADT,PCTADB/OUTPUT/ZGG,ZG,K11,K19,   00000400
     CNKER                                                                00000410
      READ (5,INPUT)                                                      00000420
      WRITE (6,INPUT)                                                     00000430
      CALL CANCEL (2)                                                     00000440
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      INNT= 1                                                          00000450
      INNB= 1                                                          00000460
      NCHDT= 20                                                        00000470
      NPRT= 4                                                          00000480
      NPL= 20                                                          00000490
      NFV= 0                                                           00000500
      EG= 1.0D-12                                                      00000510
      EGS= EG*EG                                                       00000520
      PI= 3.14159265358979300                                          00000530
      PI2= 2.0D0*PI                                                    00000540
      AT= THETAT*PI/180.0D0                                            00000550
      ZT= DCMPLX(DCOS(AT),DSIN(AT))                                    00000560
      ZTB= DCONJG(ZT)                                                  00000570
      AB= THETAB*PI/180.0D0                                            00000580
      ZB= DCMPLX(DCOS(AB),DSIN(AB))                                    00000590
      ZBB= DCONJG(ZB)                                                  00000600
      B= AT-PI/3.0D0                                                   00000610
      Z1= ZT+PCTADT*DCMPLX(DCOS(B),DSIN(B))                            00000620
      Z1B= DCONJG(Z1)                                                  00000630
      B= AB+PI/3.0D0                                                   00000640
      Z2= ZB+PCTADB*DCMPLX(DCOS(B),DSIN(B))                            00000650
      Z2B= DCONJG(Z2)                                                  00000660
C                                                                      00000670
CALCULATE CIRCUMFERENTIAL CIRCLE POINTS FOR PLOTTING PURPOSES.         00000680
C                                                                      00000690
      NS= 72                                                           00000700
      S= 2.0*PI/NS                                                     00000710
      SA= 0.0                                                          00000720
      DO 101 I= 1,NS                                                   00000730
      SA= SA+S                                                         00000740
      ZCP(I)= CMPLX(COS(SA),SIN(SA))                                   00000750
  101 CONTINUE                                                         00000760
      ZCP(NS+1)= (-1.0, 2.5)                                           00000770
      ZCP(NS+2)= (-1.0,-2.5)                                           00000780
      ZCP(NS+3)= ( 4.0, 0.0)                                           00000790
C                                                                      00000800
C  INITIALIZE ARRAYS.                                                  00000810
C                                                                      00000820
      DO 102 I= 1,NFVT                                                 00000830
      ZF(I)= 0.0D0                                                     00000840
      GF(I)= 0.0D0                                                     00000850
      CV(I)= 0.0D0                                                     00000860
  102 CONTINUE                                                         00000870
      DG1MAX= 0.0D0                                                    00000880
      DG2MAX= 0.0D0                                                    00000890
      DG1MIN= 0.0D0                                                    00000900
      DG2MIN= 0.0D0                                                    00000910
      T(1)= 0.0                                                        00000920
```

```
      DT= 2.0D-2                                                        00000930
C*******************  BEGINNING OF MAIN DO LOOP.  **********************00000940
      DO 131 IT= 1,NIT                                                  00000950
      U= 1.0D0                                                          00000960
      DUDT= 0.0D0                                                       00000970
      IF (IT.GT.NCHDT) DT= 0.05D0                                       00000980
  143 K1= (ZT-Z1)*(ZT*Z1B-1.0D0)/(1.0D0-Z1*Z1B)                         00000990
      K2= (Z2*Z2B-1.0D0)/((ZT-Z2)*(ZT*Z2B-1.0D0))                       00001000
      K3= (ZB-Z2)*(ZB*Z2B-1.0D0)/(1.0D0-Z2*Z2B)                         00001010
      K4= (Z1*Z1B-1.0D0)/((ZB-Z1)*(ZB*Z1B-1.0D0))                       00001020
      K7 = (0.0D0,1.0D0)*U*(1.0D0-(1.0D0/(ZT*ZT)))                      00001030
      K15= (0.0D0,1.0D0)*U*(1.0D0-(1.0D0/(ZB*ZB)))                      00001040
      CALL K1119 (ZT,K11)                                               00001050
      CALL K1119 (ZB,K19)                                               00001060
C                                                                       00001070
CALCULATE GROWING VORTEX CIRCULATION.                                   00001080
C                                                                       00001090
      ZAA(1)= 1.0D0                                                     00001100
      ZAA(2)= -(K4*K3)                                                  00001110
      ZAA(3)= -(K1*K2)                                                  00001120
      ZAA(4)= 1.0D0                                                     00001130
      ZCC(1)= K1*(K7+(K11/PI2))                                         00001140
      ZCC(2)= K3*(K15+(K19/PI2))                                        00001150
      AC(1)= ZCC(1)                                                     00001160
      AC(3)= DUM(2,1)                                                   00001170
      AC(2)= ZCC(2)                                                     00001180
      AC(4)= DUM(2,2)                                                   00001190
      CALL MATCON (ZAA,2,AA)                                            00001200
      CALL INVERT (4,EG,DS1AA,DS1AX,NKER(1),4)                          00001210
      CALL DMPRD (AX,AC,AG,4,4,1)                                       00001220
      IF (DABS(AG(3)).LE.EG) AG(3)= 0.0D0                               00001230
      IF (DABS(AG(4)).LE.EG) AG(4)= 0.0D0                               00001240
      ZGG(1)= DCMPLX(AG(1),AG(3))*PI2                                   00001250
      ZGG(2)= DCMPLX(AG(2),AG(4))*PI2                                   00001260
      GG1= 1.05D0*ZGG(1)                                                00001270
      GG2= ZGG(2)                                                       00001280
      K5 = ((ZT*Z1B-1.0D0)/(1.0D0-Z1*Z1B))**2                           00001290
      K6 =((ZT-Z1)/(1.0D0-Z1*Z1B))**2                                   00001300
      K8 = (0.0D0,1.0D0)*DUDT*(1.0D0-(1.0D0/(ZT*ZT)))                   00001310
      K9 = 1.0D0/((ZT-Z2)**2)                                           00001320
      K10= 1.0D0/((ZT*Z2B-1.0D0)**2)                                    00001330
      K12= K7+((GG2*K2+K11)/PI2)                                        00001340
      K13= ((ZB*Z2B-1.0D0)/(1.0D0-Z2*Z2B))**2                           00001350
      K14=((ZB-Z2)/(1.0D0-Z2*Z2B))**2                                   00001360
      K16= (0.0D0,1.0D0)*DUDT*(1.0D0-(1.0D0/(ZB*ZB)))                   00001370
      K17= 1.0D0/((ZB-Z1)**2)                                           00001380
      K18= 1.0D0/((ZB*Z1B-1.0D0)**2)                                    00001390
      K20= K15+((GG1*K4+K19)/PI2)                                       00001400
```

```
C                                                                               00001410
CALCULATION OF TIME DERIVATIVES.                                                00001420
C                                                                               00001430
      CALL ZAARRY                                                               00001440
C                                                                               00001450
CALCULATE ZC(I)                                                                 00001460
C                                                                               00001470
      DO 110 I=1,2                                                              00001480
      IF (I.EQ.1) ZZ= ZT                                                        00001490
      IF (I.EQ.2) ZZ= ZB                                                        00001500
      K= 0.0D0                                                                  00001510
      IF (NFV) 109,109,103                                                      00001520
  103 DO 108 J= 1,NFV                                                           00001530
      ZS= 0.0D0                                                                 00001540
      ZD= ZF(J)                                                                 00001550
      IF ((CDABS(ZD   ).LE.EG) .OR.(DABS(GF(J)).LE.EG)) GO TO 108               00001560
      CALL DCVELP (ZD,CVV)                                                      00001570
      ZR= (ZZ-ZD   )**2                                                         00001580
      IF (CDABS(ZR)-EGS) 105,105,104                                            00001590
  104 ZS= CVV/ZR                                                                00001600
  105 ZD= DCONJG (ZD)                                                           00001610
      ZR= (ZZ*ZD-1.0D0)**2                                                      00001620
      IF (CDABS(ZR)-EGS) 107,107,106                                            00001630
  106 CVV= DCONJG(CVV)                                                          00001640
      ZS= ZS+CVV/ZR                                                             00001650
  107 K= K+GF(J)*ZS                                                             00001660
  108 CONTINUE                                                                  00001670
  109 IF (I.EQ.1) ZC(I)= K1*((K/PI2)+K8)                                        00001680
      IF (I.EQ.2) ZC(I)= K3*((K/PI2)+K16)                                       00001690
  110 CONTINUE                                                                  00001700
  112 CALL DCVELP (Z1,ZC(3))                                                    00001710
      ZC(4)= DCONJG(ZC(3))                                                      00001720
      CALL DCVELP (Z2,ZC(5))                                                    00001730
      ZC(6)= DCONJG(ZC(5))                                                      00001740
      DO 135 I= 1,6                                                             00001750
      AC(I)= ZC(I)                                                              00001760
      AC(I+6)= DUM(2,I)                                                         00001770
  135 CONTINUE                                                                  00001780
      CALL MATCON (ZA,6,AA)                                                     00001790
      CALL INVERT (12,EG,DS2AA,DS2AX,NKER(2),12)                                00001800
      CALL DMPRD (AX,AC,AG,12,12,1)                                             00001810
      IF (DABS(AG(7)).LE.EG) AG(7)= 0.0D0                                       00001820
      IF (DABS(AG(8)).LE.EG) AG(8)= 0.0D0                                       00001830
      DO 136 I= 1,6                                                             00001840
      ZC(I)= DCMPLX(AG(I),AG(I+6))                                              00001850
  136 CONTINUE                                                                  00001860
      DG1(IT)= ZG(1)                                                            00001870
      DG2(IT)= ZG(2)                                                            00001880
```

```
C                                                                       00001890
CALCULATE DRAG AND LIFT COIFFICIENTS.                                   00001900
C                                                                       00001910
  113 K= PI2*DUDT                                                       00001920
      ZS= 0.0D0                                                         00001930
      IF (INNT.EQ.1) K= K+(0.0D0,1.0D0)*(DG1(IT)*(Z1-(1.0D0/Z1B))       00001940
     C                   +GG1*(ZG(3)+DCONJG(ZG(3))/(DCONJG(Z1)**2)))    00001950
      IF (INNB.EQ.1) K= K+(0.0D0,1.0D0)*(DG2(IT)*(Z2-(1.0D0/Z2B))       00001960
     C                   +GG2*(ZG(5)+DCONJG(ZG(5))/(DCONJG(Z2)**2)))    00001970
      IF (NFV) 117,117,115                                              00001980
  115 DO 116 I=1,NFV                                                    00001990
      IF ((CDABS(ZF(I)).LE.EG).OR.(DABS(GF(I)).LE.EG)) CV(I)= 0.0D0     00002000
      IF ((CDABS(ZF(I)).LE.EG).OR.(DABS(GF(I)).LE.EG)) GO TO 116        00002010
      CALL DCVELP (ZF(I),CVV)                                           00002020
      CV(I)= CVV                                                        00002030
      ZS= ZS+ (0.0D0,1.0D0)*GF(I)*(CVV+DCONJG(CVV)/(DCONJG(ZF(I))**2))  00002040
  116 CONTINUE                                                          00002050
  117 CDCL(IT)= K+ZS                                                    00002060
C                                                                       00002070
C WRITE AND PLOT.                                                       00002080
C                                                                       00002090
      IF (IT.LE.15) GO TO 141                                           00002100
      IF ((IT/NPRT)*NPRT-IT) 140,141,141                                00002110
  141 WRITE (6,2) IT,T(IT),DT,CDCL(IT),Z1,ZG(3),ZC(3),GG1,DG1(IT),      00002120
     CTHETAT,Z2,ZG(5),ZC(5),GG2,DG2(IT),THETAB,DG1MIN,DG2MIN            00002130
      IF (NFV) 119,119,118                                              00002140
  118 J= NFV-1                                                          00002150
      WRITE (6,3) (ZF(I),CV(I),GF(I),ZF(I+1),CV(I+1),GF(I+1),I=1,J,2)   00002160
  119 WRITE (6,OUTPUT)                                                  00002170
  140 IF ((IT/NPL)*NPL-IT) 125,120,120                                  00002180
  120 NPAIR= NS+3                                                       00002190
      DO 121 I= 1,NPAIR                                                 00002200
      XP(I)= ZCP(I)                                                     00002210
      YP(I)= AIMAG(ZCP(I))                                              00002220
  121 CONTINUE                                                          00002230
      N= 1                                                              00002240
      CALL PLOTP (XP,YP,NPAIR,N)                                        00002250
      IF (NFV) 124,124,122                                              00002260
  122 NPAIR= NFV                                                        00002270
      DO 123 I= 1,NFV                                                   00002280
      XP(I)= ZF(I)                                                      00002290
      ZZ8= ZF(I)                                                        00002300
      YP(I)= AIMAG(ZZ8)                                                 00002310
  123 CONTINUE                                                          00002320
      N= 2                                                              00002330
      CALL PLOTP (XP,YP,NPAIR,N)                                        00002340
  124 NPAIR=2                                                           00002350
      ZZ8= Z1                                                           00002360
```

```
      XP(1)= ZZ8                                                        00002370
      YP(1)= AIMAG(ZZ8)                                                 00002380
      ZZ8= Z2                                                           00002390
      XP(2)= ZZ8                                                        00002400
      YP(2)= AIMAG(ZZ8)                                                 00002410
      N= 3                                                              00002420
      WRITE (6,1)                                                       00002430
      CALL PLOTP (XP,YP,NPAIR,N)                                        00002440
      WRITE (6,8) IT                                                    00002450
      WRITE (6,1)                                                       00002460
C                                                                       00002470
C ADVANCE ALL PARAMETERS.                                               00002480
C                                                                       00002490
  125 IF (NFV.GE.NFVT) GO TO 132                                        00002500
      T(IT+1)= T(IT)+DT                                                 00002510
      IF (NFV) 137,137,126                                              00002520
  126 DO 127 I= 1,NFV                                                   00002530
      ZF(I)= ZF(I)+CV(I)*DT                                             00002540
  127 CONTINUE                                                          00002550
  137 D= ZG(1)                                                          00002560
      CALL RUNKUT (Z1,ZT,DT,D,GG1,ZC(3),ZZ)                             00002570
      Z1= ZZ                                                            00002580
      Z1B= DCONJG(Z1)                                                   00002590
      D= ZG(2)                                                          00002600
      CALL RUNKUT (Z2,ZB,DT,D,GG2,ZC(5),ZZ)                             00002610
      Z2= ZZ                                                            00002620
      Z2B= DCONJG(Z2)                                                   00002630
C                                                                       00002640
CREATE FREE VORTS. IF APPROPRIATE.                                      00002650
C                                                                       00002660
      IF (IT.LE.NCHDT) GO TO 114                                        00002670
      B= (-ZG(1))                                                       00002680
      IF (B.GT.DG1MAX) DG1MAX= B                                        00002690
      IF (B.EQ.DG1MAX) DG1MIN= B                                        00002700
      IF ((B.LT.DG1MAX).AND.(B.LT.DG1MIN)) DG1MIN= B                    00002710
      IF ((B.LT.DG1MAX).AND.(B.GT.DG1MIN)) GO TO 111                    00002720
      GO TO 138                                                         00002730
  111 NFV= NFV+1                                                        00002740
      INNT= 0                                                           00002750
      ZF(NFV)= Z1                                                       00002760
      GF(NFV)= GG1                                                      00002770
      DG1MAX= 0.0D0                                                     00002780
      DG1MIN= 0.0D0                                                     00002790
      DT = 2.0D-2                                                       00002800
      IF (NFV.GE.NFVT) GO TO 114                                        00002810
  138 B= ZG(2)                                                          00002820
      IF (B.GT.DG2MAX) DG2MAX= B                                        00002830
      IF (B.EQ.DG2MAX) DG2MIN= B                                        00002840
```

```
      IF ((B.LT.DG2MAX).AND.(B.LT.DG2MIN)) DG2MIN= B                    00002850
      IF ((B.LT.DG2MAX).AND.(B.GT.DG2MIN)) GO TO 145                    00002860
      GO TO 114                                                         00002870
  145 NFV= NFV+1                                                        00002880
      INNB= 0                                                           00002890
      ZF(NFV)= Z2                                                       00002900
      GF(NFV)= GG2                                                      00002910
      DG2MAX= 0.0D0                                                     00002920
      DG2MIN= 0.0D0                                                     00002930
      DT = 2.0D-2                                                       00002940
  114 IF ((INNT.EQ.0).OR.(INNB.EQ.0)) NCHDT= IT+20                      00002950
C                                                                       00002960
C  INTRODUCE NEW GROWING VORTS. IF APPROPRIATE.                         00002970
C                                                                       00002980
      IF (INNT) 128,128,129                                             00002990
  128 INNT= 1                                                           00003000
      AT= THETAT*PI/180.0D0                                             00003010
      ZT= DCMPLX(DCOS(AT),DSIN(AT))                                     00003020
      ZTB= DCONJG(ZT)                                                   00003030
      B= AT-PI/3.0D0                                                    00003040
      Z1= ZT+PCTADT*DCMPLX(DCOS(B),DSIN(B))                             00003050
      Z1B= DCONJG(Z1)                                                   00003060
  129 IF (INNB) 130,130,131                                             00003070
  130 INNB= 1                                                           00003080
      AB= THETAB*PI/180.0D0                                             00003090
      ZB= DCMPLX(DCOS(AB),DSIN(AB))                                     00003100
      ZBB= DCONJG(ZB)                                                   00003110
      B= AB+PI/3.0D0                                                    00003120
      Z2= ZB+PCTADB*DCMPLX(DCOS(B),DSIN(B))                             00003130
      Z2B= DCONJG(Z2)                                                   00003140
  131 CONTINUE                                                          00003150
C**********   END OF MAIN DO LOOP  ************************************00003160
  132 WRITE (6,1)                                                       00003170
      IF (IT.GT.NIT) IT= IT-1                                           00003180
      WRITE (6,7) (I,T(I),CDCL(I),DG1(I),DG2(I),I=1,IT)                 00003190
      DO 133 I= 1,IT                                                    00003200
      YP(I)= CDCL(I)                                                    00003210
      IF (ABS(YP(I)).GT.2.4) YP(I)= 0.0                                 00003220
  133 CONTINUE                                                          00003230
      N= IT+1                                                           00003240
      YP(N)= 2.4                                                        00003250
      T(N)= 0.0                                                         00003260
      WRITE (6,1)                                                       00003270
      CALL PLOTP (T,YP,N,0)                                             00003280
      WRITE (6,4)                                                       00003290
      DO 134 I= 1,IT                                                    00003300
      YP(I)= AIMAG(CDCL(I))                                             00003310
      IF (ABS(YP(I)).GT.2.4) YP(I)= 0.0                                 00003320
```

```
  134 CONTINUE                                                          00003330
      WRITE (6,1)                                                       00003340
      CALL PLOTP (T,YP,N,0)                                             00003350
      WRITE (6,5)                                                       00003360
      DG1(N)= 0.0                                                       00003370
      DO 144 I= 1,N                                                     00003380
      DG1(I)= (-DG1(I))                                                 00003390
      IF (ABS(DG1(I)).GT.4.0) DG1(I)= 0.0                               00003400
      IF (ABS(DG2(I)).GT.4.0) DG2(I)= 0.0                               00003410
  144 CONTINUE                                                          00003420
      CALL PLOTP (T,DG1,N ,1)                                           00003430
      WRITE (6,1)                                                       00003440
      DG2(N)= 0.0                                                       00003450
      CALL PLOTP (T,DG2,N ,3)                                           00003460
      WRITE (6,6)                                                       00003470
      RETURN                                                            00003480
    1 FORMAT (1H1)                                                      00003490
    2 FORMAT (////,3X,'IT = ',I4,4X,'TIME = ',F7.3,4X,'DELTA TIME = ',  00003500
     CF9.5,                                                             00003510
     C4X,'CDCL =',2F7.3,                                                00003520
     C//,T41,'CORE',T57,'TRUE CORE',T79,'DW/DZ',T95,'CORE',T106,'D-GAMMA00003530
     C',T118,'FEEDING',/,T39,'POSITION',T57,'VELOCITY',T78,'VELOCITY',  00003540
     CT93, 'STRENGTH',T108,'DT',T116,'SHEET ORIGIN',/,T37,'X',T46,'IY',  00003550
     CT54,'DX/DT',T63,'I DY/DT',T77,'U',T86,'IV',T118,'(DEGREES)',//,    00003560
     C6X,'UPPER FEEDING SHEET',6X,2(F7.3,3X),4(F8.4,2X),1X,F7.3,4X,F8.3,00003570
     C4X,F7.2,/,                                                        00003580
     C6X,'LOWER FEEDING SHEET',6X,2(F7.3,3X),4(F8.4,2X),1X,F7.3,4X,F8.3,00003590
     C4X,F7.2,///,T11,'NOTE: THE VORTEX SHEETS ARE CUT, FREEING THE CORE00003600
     CS, WHEN ABS(D-GAMMA/DT) IS:  TOP = ',F8.3,', BOTTOM = ',F8.3,////)00003610
    3 FORMAT (T8,'FREE VORTEX CORES.',//,T19,'ORIGINATING FROM UPPER VOR00003620
     CTEX SHEET',T85,'ORIGINATING FROM LOWER VORTEX SHEET',//,2(7X,'POSI00003630
     CTION',                                                            00003640
     C   15X,'VELOCITY',12X,'STRENGTH',8X),/,2(6X,'X',8X,'IY',11X,'U',8X00003650
     C,'IV',27X),/,2(3X,2(F7.3,2X),4X,2(F7.3,2X),7X,F7.3,9X))           00003660
    4 FORMAT (///,3X,'DRAG COEFFICIENT VERSUS TIME.')                   00003670
    5 FORMAT (///,3X,'LIFT COEFFICIENT VERSUS TIME.')                   00003680
    6 FORMAT (///,3X,'D-GAMMA/DT VERSUS TIME. UPPER SHEET .....',/,     00003690
     CT28,'LOWER SHEET ++++++',//,3X,'NOTE: THE UPPER SHEET IS PLOTTED A00003700
     CS THE NEGATIVE OF ITS TRUE VALUE.')                               00003710
    7 FORMAT (///,T5,'PASS NO.',T13,'TIME',T29,'DRAG',T40,'LIFT',T51,   00003720
     C'DG/DT',T63,'DG/DT',/,T51,'(TOP)',T62,'(BOTTOM)',//,              00003730
     C(3X,I4,8X,3(F6.3,5X),2(F7.3,5X),60X))                             00003740
    8 FORMAT (/////,10X,'VORTEX POSITION PLOT FOR IT = ',I4)            00003750
      END                                                               00003760
```

```
      SUBROUTINE DCVELP (Z,CV)                                          00003770
C*****************************************************************    00003780
C                                                                       00003790
C     PURPOSE: TO CALCULATE THE FREE CONVECTIVE VELOCITY AT ANY POSITION,00003800
C              ON OR OUTSIDE THE CYLINDER.                              00003810
C                                                                       00003820
C     PARAMETERS: Z = INPUT POSITION; COMPLEX*16                        00003830
C                 CV = OUTPUT LOCAL CONVECTIVE VELOCITY; COMPLEX*16     00003840
C                                                                       00003850
C*****************************************************************    00003860
      IMPLICIT INTEGER(I,J,L,M,N,P,Q),REAL*8(A,B,D-H,O,R-Y),            00003870
     CCOMPLEX*16(C,K,Z)                                                 00003880
      REAL*8 PI,PI2                                                     00003890
      COMMON /ALPHA/ZF(20),GF(20),EG,EGS,NFV                            00003900
      COMMON /BETA/Z1,Z1B,Z2,Z2B,U,GG1,GG2                              00003910
      PI= 3.1415926535897930                                            00003920
      PI2= 2.0D0*PI                                                     00003930
      K= 0.0D0                                                          00003940
      IF (CDABS(Z)-EG) 114,114,101                                      00003950
C                                                                       00003960
CALCULATION FOR FREE VORTS.                                             00003970
C                                                                       00003980
  101 IF (NFV) 108,108,102                                              00003990
  102 DO 107 I= 1,NFV                                                   00004000
      ZS= 0.0D0                                                         00004010
      IF ((CDABS(ZF(I)).LE.EG).OR.(DABS(GF(I)).LE.EG)) GO TO 107        00004020
      ZR= Z-ZF(I)                                                       00004030
      IF (CDABS(ZR)-EG) 104,104,103                                     00004040
  103 ZS= 1.0D0/ZR                                                      00004050
  104 ZR= Z-(1.0D0/DCONJG(ZF(I)))                                       00004060
      IF (CDABS(ZR)-EG) 106,106,105                                     00004070
  105 ZS= ZS-1.0D0/ZR                                                   00004080
  106 K= K+GF(I)*ZS                                                     00004090
  107 CONTINUE                                                          00004100
C                                                                       00004110
CALCULATION FOR GROWING VORTS.                                          00004120
C                                                                       00004130
  108 DO 113 I= 1,2                                                     00004140
      IF (I.EQ.1) ZZ= Z1                                                00004150
      IF (I.EQ.1) G= GG1                                                00004160
      IF (I.EQ.2) ZZ= Z2                                                00004170
      IF (I.EQ.2) G= GG2                                                00004180
      ZS= 0.0D0                                                         00004190
      IF ((CDABS(ZZ).LE.EG).OR.(DABS(G).LE.EG)) GO TO 113               00004200
      ZR= Z-ZZ                                                          00004210
      IF (CDABS(ZR)-EG) 110,110,109                                     00004220
```

```
109 ZS= 1.CDC/ZR                                                                00004230
110 ZR= Z-(1.CDO/DCONJG(ZZ))                                                    00004240
    IF (CDABS(ZR)-EG) 112,112,111                                               00004250
111 ZS= ZS-(1.CDO/ZR)                                                           00004260
112 K= K+G*ZS                                                                   00004270
113 CONTINUE                                                                    00004280
    ZS= U*((1.0DO/(Z**2))-1.0DO)+((0.CDO,1.CDO)*K/PI2)                          00004290
    CV= (-DCONJG(ZS))                                                           00004300
    GO TO 115                                                                   00004310
114 WRITE (6,1) Z                                                               00004320
115 RETURN                                                                      00004330
  1 FORMAT (1H1,3X,'**** ATTEMPT TO CALCULATE CONVECTIVE VELOCITY AT C00004340
   CENTER OF CYLINDER, ',//,3X,'Z = ',2F23.16)                                  00004350
    END                                                                         00004360
```

```
      SUBROUTINE  DMPRD (A,B,R,N,M,L)                                   00004370
C*****************************************************************    00004380
C                                                                       00004390
C     PURPOSE: TO MULTIPLY TWO COMPLEX*16 GENERAL MATRICIES TOGETHER.   00004400
C                                                                       00004410
C     PARAMETERS: A = FIRST INPUT VECTOR.                               00004420
C                 B = SECOND INPUT VECTOR.                              00004430
C                 R = OUTPUT VECTOR.                                    00004440
C                 N = # ROWS IN MATRIX A, REAL*4                        00004450
C                 M = # OF COLUMNS IN A AND ROWS IN B, INTEGER*4        00004460
C                 L = # OF COLUMNS IN B, INTEGER*4                      00004470
C                                                                       00004480
C     FORMULA:                                                          00004490
C                                                                       00004500
C                       |A||B|=|R|                                      00004510
C                                                                       00004520
C*****************************************************************    00004530
      REAL*8      A(1),B(1),R(1)                                        00004540
      IR= 0                                                             00004550
      IK= -M                                                            00004560
      DO 10 K= 1,L                                                      00004570
      IK= IK+M                                                          00004580
      DO 10 J= 1,N                                                      00004590
      IR= IR+1                                                          00004600
      JI= J-N                                                           00004610
      IB= IK                                                            00004620
      R(IR)= 0.0D0                                                      00004630
      DO 10 I= 1,M                                                      00004640
      JI= JI+N                                                          00004650
      IB= IB+1                                                          00004660
   10 R(IR)= R(IR)+A(JI)*B(IB)                                          00004670
      RETURN                                                            00004680
      END                                                               00004690
```

```
      SUBROUTINE INVERT(N,EP,A,X,KER,NACT)                              00004700
C*****************************************************************     00004710
C                                                                       00004720
C     PURPOSE: INVERT A REAL*8 DOUBLE DIMENSIONED ARRAY BY THE GAUSSIAN-00004730
C              ELIMINATION WITH BACK SUBSTITUTION METHOD.               00004740
C                                                                       00004750
C     PARAMETERS: N = ORDER OF MATRIX A.                                00004760
C                 EP = DIFFERENCE ERROR (1.0D-10 FOR DOUBLE PRECISION). 00004770
C                 A = DOUBLE DIMENSION REAL*8 INPUT MATRIX.             00004780
C                 X = INVERSE OF MATRIX A, OUTPUT, REAL*8.              00004790
C                 KER = IF KER IS RETURNED EQUAL TO 2, MATRIX A IS      00004800
C                       SINGULAR.                                       00004810
C                 NACT = NUMERICAL VALUE OF N.                          00004820
C                                                                       00004830
C*****************************************************************     00004840
      IMPLICIT REAL*8(A-H,O-Z)                                          00004850
      DIMENSION A(NACT,NACT),X(NACT,NACT)                               00004860
      DO 1 I=1,N                                                        00004870
      DO 1 J=1,N                                                        00004880
    1 X(I,J)=0.0D0                                                      00004890
      DO 2 K=1,N                                                        00004900
    2 X(K,K)=1.0D0                                                      00004910
   10 DO 34 L=1,N                                                       00004920
      KP=0                                                              00004930
      Z=0.0D0                                                           00004940
      DO 12 K=L,N                                                       00004950
      IF(Z-DABS(A(K,L)))11,12,12                                        00004960
   11 Z=DABS(A(K,L))                                                    00004970
      KP=K                                                              00004980
   12 CONTINUE                                                          00004990
      IF(L-KP)13,20,20                                                  00005000
   13 DO 14 J=L,N                                                       00005010
      Z=A(L,J)                                                          00005020
      A(L,J)=A(KP,J)                                                    00005030
   14 A(KP,J)=Z                                                         00005040
      DO 15 J=1,N                                                       00005050
      Z=X(L,J)                                                          00005060
      X(L,J)=X(KP,J)                                                    00005070
   15 X(KP,J)=Z                                                         00005080
   20 IF(DABS(A(L,L))-EP)50,50,30                                       00005090
   30 IF(L-N)31,34,34                                                   00005100
   31 LP1=L+1                                                           00005110
      DO 36 K=LP1,N                                                     00005120
      IF(A(K,L))32,36,32                                                00005130
   32 RATIO=A(K,L)/A(L,L)                                               00005140
      DO 33 J=LP1,N                                                     00005150
```

```
33 A(K,J)=A(K,J)-RATIO*A(L,J)                                          00005160
   DO 35 J=1,N                                                         00005170
35 X(K,J)=X(K,J)-RATIO*X(L,J)                                          00005180
36 CONTINUE                                                            00005190
34 CONTINUE                                                            00005200
40 DO 43 I=1,N                                                         00005210
   II=N+1-I                                                            00005220
   DO 43 J=1,N                                                         00005230
   S=0.0D0                                                             00005240
   IF(II-N)41,43,43                                                    00005250
41 IIP1=II+1                                                           00005260
   DO 42 K=IIP1,N                                                      00005270
42 S=S+A(II,K)*X(K,J)                                                  00005280
43 X(II,J)=(X(II,J)-S)/A(II,II)                                        00005290
   KER=1                                                               00005300
   GO TO 75                                                            00005310
50 KER=2                                                               00005320
75 CONTINUE                                                            00005330
   RETURN                                                              00005340
   END                                                                 00005350
```

```
      SUBROUTINE K1119 (Z,K)                                                    00005360
C*****************************************************************************00005370
C                                                                               00005380
C     PURPOSE: TO CALCULATE THE CONSTANTS K11 AND K19.                          00005390
C                                                                               00005400
C     PARAMETERS:  Z = INPUT, TOP OR BOTTOM FEEDING SHEET; COMPLEX*16           00005410
C                  K = OUTPUT, K11 OR K19 DEPENDING ON Z; COMPLEX*16            00005420
C                                                                               00005430
C*****************************************************************************00005440
      IMPLICIT INTEGER(I,J,L,M,N,P,Q),REAL*8(A,B,D-H,O,R-Y),                    00005450
     CCOMPLEX*16(C,K,Z)                                                         00005460
      COMMON /ALPHA/ZF(20),GF(20),EG,EGS,NFV                                    00005470
      K= 0.0D0                                                                  00005480
      IF (NFV) 109,109,101                                                      00005490
  101 IF (CDABS(Z)-EG) 108,108,102                                              00005500
  102 DO 107 I= 1,NFV                                                           00005510
      ZS= 0.0D0                                                                 00005520
      IF ((CDABS(ZF(I)).LE.EG).OR.(DABS(GF(I)).LE.EG)) GO TO 107                00005530
      ZR= Z-ZF(I)                                                               00005540
      IF (CDABS(ZR)-EG) 104,104,103                                             00005550
  103 ZS= 1.0D0/ZR                                                              00005560
  104 ZR= Z-(1.0D0/DCONJG(ZF(I)))                                               00005570
      IF (CDABS(ZR)-EG) 106,106,105                                             00005580
  105 ZS= ZS-1.0D0/ZR                                                           00005590
  106 K= K+GF(I)*ZS                                                             00005600
  107 CONTINUE                                                                  00005610
      GO TO 109                                                                 00005620
  108 WRITE (6,1) Z                                                             00005630
  109 RETURN                                                                    00005640
    1 FORMAT (1H1,3X,'**** ATTEMPT TO CALCULATE K1119 FOR A ZB OR ZT AT         00005650
     CCENTER OF CYLINDER',//,3X,'ZB OR ZT = ',2F23.16)                          00005660
      END                                                                       00005670
```

```
      SUBROUTINE MATCON (B,N,G)                                          00005680
C*****************************************************************     00005690
C                                                                       00005700
C     PURPOSE: TO CONVERT A COMPLEX*16 MATRIX VECTOR INTO A REAL*8      00005710
C              PARTITIONED MATRIX VECTOR OF 2N ORDER, READY FOR INVERSION00005720
C              USING SUBROUTINE INVERT.                                 00005730
C                                                                       00005740
C              IF |A| = |B| + I|C|, THEN:                               00005750
C                                                                       00005760
C                                                                       00005770
C                                  |G| = | |B|  -|C| |                  00005780
C                                        |        |                     00005790
C                                        | |C|   |B| |                  00005800
C                                                                       00005810
C                                                                       00005820
C     PARAMETERS: A = INPUT VECTOR REPRESENTING A GENERAL SQUARE MATRIX. 00005830
C                     COMPLEX*16                                        00005840
C                 N = INPUT ORDER OF MATRIX A, INTEGER*4.               00005850
C                 G = OUTPUT VECTOR, REPRESENTS A SQUARE MATRIX. REAL*8 00005860
C                     MUST HAVE  (2*N)**2 ELEMENTS.                     00005870
C                                                                       00005880
C*****************************************************************     00005890
      REAL*8 G(1),DUM(2,36)                                             00005900
      COMPLEX*8 ZZ                                                      00005910
      COMPLEX*16 A(36),B(36)                                            00005920
      EQUIVALENCE (A(1),DUM(1,1))                                       00005930
      DO 105 I= 1,36                                                    00005940
      A(I)= B(I)                                                        00005950
  105 CONTINUE                                                          00005960
      NAE= N*N                                                          00005970
C                                                                       00005980
CONVERT REAL PARTS.                                                     00005990
C                                                                       00006000
      M= 2*N*N+N                                                        00006010
      DO 102 I= 1,2                                                     00006020
      DO 101 J = 1,NAE                                                  00006030
      L= N*((J-1)/N)+(I-1)*M+J                                          00006040
      G(L)= A(J)                                                        00006050
  101 CONTINUE                                                          00006060
  102 CONTINUE                                                          00006070
C                                                                       00006080
CONVERT IMAGINARY PARTS.                                                00006090
C                                                                       00006100
      K= 2*N*(1-N)                                                      00006110
      M= 2*N*N-N                                                        00006120
      DO 104 I= 1,2                                                     00006130
```

```
      DO 103 J= 1,NAE                                                   00006140
      L= N*((J-1)/N)+I*M+K+J                                            00006150
      G(L)= DUM(2,J)*((-1.0D0)**(I+1))                                  00006160
  103 CONTINUE                                                          00006170
  104 CONTINUE                                                          00006180
      RETURN                                                            00006190
      END                                                               00006200
```

```
      SUBROUTINE RUNKUT (Z,ZO,DT,DGDT,GAMMA,U,ZNEW)                     00006210
C*****************************************************************    00006220
C                                                                      00006230
C     PURPOSE: TO FIND THE NEXT GROWING VORTEX POSITION BY INTEGRATING 00006240
C              THE FORCE EQUATION OVER THE NEXT TIME STEP USING THE    00006250
C              RUNGA-KUTTA APPROXIMATION.                              00006260
C                                                                      00006270
C     PARAMETERS: Z = PRESENT GROWING VORTEX POSITION. COMPLEX*16      00006280
C                 DT = TIME STEP. REAL*8                               00006290
C                 DGDT = D-GAMMA/DT OF GROWING VORTEX, CONSIDERED      00006300
C                        CONSTANT OVER DT. REAL*8                      00006310
C                 GAMMA = GROWING VORTEX STRENGTH AT POSITION Z. REAL*8 00006320
C                 U = VELOCITY OF VORTEX IF IT WERE FREE (U+IV),       00006330
C                     CONSIDERED CONSTANT OVER DT. COMPLEX*16          00006340
C                 ZO = FEEDING SHEET ORIGIN FOR VORTEX IN QUESTION.    00006350
C                     COMPLEX*16                                       00006360
C                                                                      00006370
C*****************************************************************    00006380
      COMPLEX*16 Z,ZO,U,ZNEW,K1,K2,K3,K4                               00006390
      REAL*8 DT,DGDT,GAMMA                                             00006400
      DGDT= DGDT/GAMMA                                                 00006410
      K1= DT*((ZO-Z)*DGDT+U)                                           00006420
      K2= DT*((ZO-Z-(K1/2.0D0))*DGDT+U)                                00006430
      K3= DT*((ZO-Z-(K2/2.0D0))*DGDT+U)                                00006440
      K4= DT*((ZO-Z-K3)*DGDT+U)                                        00006450
      ZNEW= (K1+2.0D0*(K2+K3)+K4)/6.0D0                                00006460
      ZNEW= ZNEW+Z                                                     00006470
      RETURN                                                           00006480
      END                                                              00006490
```

```
      SUBROUTINE ZAARRY                                                 00006500
C*****************************************************************    00006510
C                                                                       00006520
C     PURPOSE: TO DECREASE THE SIZE OF THE MAIN PROGRAM SO THAT THE     00006530
C              FORTRAN IV H COMPILER CAN BE UTILIZED.                   00006540
C                                                                       00006550
C     PARAMETERS: SAME AS THE MAIN PROGRAM. COMMON STATEMENTS TRANSFER  00006560
C                 THE VALUES.                                           00006570
C                                                                       00006580
C*****************************************************************    00006590
      IMPLICIT COMPLEX*16(K,Z),REAL*8(P,U,G)                            00006600
      COMMON /BETA/Z1,Z1B,Z2,Z2B,U,GG1,GG2                              00006610
      COMMON /CHARLE/ZA(36),K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13, 00006620
     CK14,K15,K16,K17,K18,K19,K20,ZT,ZTB,ZB,ZBB,PI,PI2                  00006630
      ZA(1 )= 1.0D0/PI2                                                 00006640
      ZA(2 )= -K4*K3/PI2                                                00006650
      ZA(3 )= (Z1-ZT)/GG1                                               00006660
      ZA(4 )= (Z1B-ZTB)/GG1                                             00006670
      ZA(5 )= 0.0D0                                                     00006680
      ZA(6 )= 0.0D0                                                     00006690
      ZA(7 )= -K1*K2/PI2                                                00006700
      ZA(8 )= 1.0D0/PI2                                                 00006710
      ZA(9 )= 0.0D0                                                     00006720
      ZA(10)= 0.0D0                                                     00006730
      ZA(11)= (Z2-ZB)/GG2                                               00006740
      ZA(12)= (Z2B-ZBB)/GG2                                             00006750
      ZA(13)= -K12*K5                                                   00006760
      ZA(14)= -GG1*K3*K17/PI2                                           00006770
      ZA(15)= 1.0D0                                                     00006780
      ZA(16)= 0.0D0                                                     00006790
      ZA(17)= 0.0D0                                                     00006800
      ZA(18)= 0.0D0                                                     00006810
      ZA(19)= -K12*K6                                                   00006820
      ZA(20)= -GG1*K3*K18/PI2                                           00006830
      ZA(21)= 0.0D0                                                     00006840
      ZA(22)= 1.0D0                                                     00006850
      ZA(23)= 0.0D0                                                     00006860
      ZA(24)= 0.0D0                                                     00006870
      ZA(25)= -GG2*K1*K9/PI2                                            00006880
      ZA(26)= -K20*K13                                                  00006890
      ZA(27)= 0.0D0                                                     00006900
      ZA(28)= 0.0D0                                                     00006910
      ZA(29)= 1.0D0                                                     00006920
      ZA(30)= 0.0D0                                                     00006930
      ZA(31)= -GG2*K1*K10/PI2                                           00006940
      ZA(32)= -K20*K14                                                  00006950
```

```
ZA(33) =  0.0D0                                                    00006960
ZA(34) =  0.0D0                                                    00006970
ZA(35) =  0.0D0                                                    00006980
ZA(36) =  1.0D0                                                    00006990
RETURN                                                             00007000
END                                                                00007010
```

```
C*****                                                                  00000010
C                                                                       00000020
C   THIS PROGRAM SIMMULATES A VORTEX SHEET THROUGH THE USE OF SMALL     00000030
C   STRENGTH POTENTIAL FLOW VORTICIES.                                  00000040
C                                                                       00000050
C   DEFINITION OF PARAMETERS IN NAMELIST INPUT BLOCK.                   00000060
C                                                                       00000070
C     TMAXP = MAXIMUM DIMENSIONLESS TIME TO WHICH PROGRAM IS TO RUN.    00000080
C             (REAL*4)                                                  00000090
C     NFOR  = 1 ,FORCED SYMMETRY IN PLACEMENT OF THE NASCENT VORTICIES. 00000100
C               THE CONDITIONS ON THE TOP ARE USED TO DESCRIBE THE      00000110
C               BOTTOM. (INTEGER)                                       00000120
C           = 2 ,THE NASCENT VORTICIES ARE PLACED IAW THE CONDITIONS     00000130
C               WITHIN THEIR OWN BIRTH AREAS. (INTEGER)                 00000140
C     NCOAL = 1 ,NO VORTEX COALESCENCE AND CANCELLATION TO BE DONE      00000150
C               DURING RUN. (INTEGER)                                   00000160
C           = 2 ,VORTEX PAIRS WILL BE COMBINED OR CANCELLED IF DISTANCE 00000170
C               BETWEEN THEM IS .LE. MZD. (INTEGER)                     00000180
C     NNAXI = 1 ,THERE IS NO ARTIFICIAL INDUCEMENT FOR THE VORTEX       00000190
C               SHEDDING TO BE ASYMMETRIC. (INTEGER)                    00000200
C           = 2 ,WHEN NTNAXI .LE. TP .LE. (NTNAXI+1), THE GAMMA IN      00000210
C               BIRTH AREA 2 IS DOUBLED IN ORDER TO INDUCE              00000220
C               NON-SYMMETRY INTO THE VORTEX STREET. (INTEGER)          00000230
C     NTNAXI = DIMENSIONLESS TIME AT WHICH NON-SYMMETRIC SHEDDING       00000240
C              IN BIRTH AREA 2 WILL START. WHEN TP .GT. (NTNAXI+1)      00000250
C              NORMAL GAMMAS WILL AGAIN BE USED. (INTEGER)              00000260
C     NDTP  = INTEGER TIME STEP AND VORTEX STRENGTH MULTIPLIER.         00000270
C                                                                       00000280
C*****                                                                  00000290
      IMPLICIT INTEGER(I-L,N),REAL(A,B,D-H,M,O-Y),COMPLEX(C,Z)          00000300
      REAL CLOCKR,CLOCKS,CLOCKT                                         00000310
      DIMENSION AVN(4),NVM(4),MP(4) ,VELM(4),ZCFP(51),ZCRP(61),ZCC(100),00000320
     CXP(100),YP(101),TP(101),CDCL(100),CVV(400)                        00000330
      COMMON /ALPHA/ZF(400),GK(400),EG,EGS,PI,NIT,NB,NV                 00000340
      CALL CANCEL (2)                                                   00000350
      NAMELIST/INPUT/RE,TMAXP,NFOR,NCOAL,NNAXI,NTNAXI,NDTP              00000360
      CLOCKR= ITIME(0)*.01                                              00000370
      CLOCKS= CLOCKR                                                    00000380
      READ (5,INPUT)                                                    00000390
      PI=3.1415926                                                      00000400
      DTP= 1.75*NDTP*PI/SQRT(RE)                                        00000410
      WRITE (6,INPUT)                                                   00000420
      EG= 1.0E-8                                                        00000430
      EGS= EG*EG                                                        00000440
```

```
      EV= 1.0E-4                                                        00000450
      MPA=1.75/SQRT(RE)                                                 00000460
      MZD= 6.0*MPA                                                      00000470
      MS= 1.0                                                           00000480
      NB= 4                                                             00000490
      NIT= 100                                                          00000500
      NPl= 1                                                            00000510
      NV= 0                                                             00000520
      NCL= 0                                                            00000530
      NCAN= 0                                                           00000540
C                                                                       00000550
CC  CALCULATE CIRCUMFERENTIAL CIRCLE POINTS FOR PLOTTING PURPOSES.      00000560
C                                                                       00000570
      NS= 72                                                            00000580
      S= 2.0*PI/NS                                                      00000590
      A = 0.0                                                           00000600
      DO 130 I= 1,NS                                                    00000610
      A= A+S                                                            00000620
      ZCC(I)= CMPLX(COS(A),SIN(A))                                      00000630
  130 CONTINUE                                                          00000640
      ZCC(NS+1)= (-1.0, 2.5)                                            00000650
      ZCC(NS+2)= (-1.0,-2.5)                                            00000660
      ZCC(NS+3)= ( 4.0, 0.0)                                            00000670
C                                                                       00000680
CC  INITIALIZE ARRAYS.                                                  00000690
C                                                                       00000700
      Z= (0.0,0.0)                                                      00000710
      DO 1012 I= 1,NIT                                                  00000720
      XP(I)= 0.0                                                        00000730
      YP(I)= 0.0                                                        00000740
      TP(I)= 0.0                                                        00000750
      CDCL(I)= Z                                                        00000760
 1012 CONTINUE                                                          00000770
      N= 4*NIT                                                          00000780
      DO 1011 I= 1,N                                                    00000790
      ZF(I)= Z                                                          00000800
      GK(I)= Z                                                          00000810
 1011 CONTINUE                                                          00000820
C                                                                       00000830
CC  POSITIONS ON CYLINDER, 60 TO 0 DEG. AND 135 TO 85 DEG.,TOP ONLY     00000840
CC  BOTTOM IS THE CONJUGATE OF THE TOP.                                 00000850
C                                                                       00000860
      DO 102 I=1,51                                                     00000870
      A   =(136-I)*PI/180.0                                            00000880
      ZCFP(I)= CMPLX(MS*COS(A),MS*SIN(A))                               00000890
  102 CONTINUE                                                          00000900
      DO 136 I= 1,61                                                    00000910
      A   = (61-I)*PI/180.0                                            00000920
```

```
      ZCFP(I)= CMPLX(MS*COS(A),MS*SIN(A))                                00000930
  136 CONTINUE                                                           00000940
C*****BEGINNING OF MAIN DO LOOP. ***********************************00000950
      DO 129 IT= 1,NIT                                                   00000960
      DO 101 I =1,NB                                                     00000970
      NVM(I)= 0                                                          00000980
      AVN(I)= 0.0                                                        00000990
      VELM(I)= 0.0                                                       00001000
      MP(I)= 0.0                                                         00001010
  101 CONTINUE                                                           00001020
C                                                                        00001030
C  NORMALIZED VELOCITIES AT THE FREE VORTICIES AND IMAGES.               00001040
C                                                                        00001050
      IF (NV.LE.0) GO TO 103                                             00001060
      DO 105 I=1,NB                                                      00001070
      K= 1+NIT*(I-1)                                                     00001080
      L= NV+NIT*(I-1)                                                    00001090
      DO 104 J=K,L                                                       00001100
      IF (ABS(GK(J)).LE.EG) GO TO 120                                    00001110
      Z= ZF(J)                                                           00001120
      CALL CVELP(Z,1,J,CV)                                               00001130
      CVV(J)= CV                                                         00001140
      GO TO 104                                                          00001150
  120 CVV(J)= (0.0,0.0)                                                  00001160
  104 CONTINUE                                                           00001170
  105 CONTINUE                                                           00001180
C                                                                        00001190
C  SEARCH TOP AND BOTTOM FRONT HALF FOR MAX. VELOCITY.                   00001200
C                                                                        00001210
  103 DO 112 I=1,2                                                       00001220
      VELMIN= 3.0                                                        00001230
      DO 111 J=1,51                                                      00001240
      IF (I.EQ.1) Z=ZCFP(J)                                              00001250
      IF (I.EQ.2) Z= CONJG(ZCFP(J))                                      00001260
      CALL CVELP(Z,0,0,CV)                                               00001270
      UI= REAL(CV)                                                       00001280
      A= CABS(CV)                                                        00001290
      IF ((UI.GE.0.0).AND.(A.GT.VELM(I))) GO TO 162                      00001300
      IF ((UI.GE.0.0).AND.(A.LE.VELM(I))) GO TO 112                      00001310
      GO TO 111                                                          00001320
  162 VELM(I)= A                                                         00001330
      NVM(I)= ((-1)**(I+1))*(136-J)                                      00001340
  111 CONTINUE                                                           00001350
  112 CONTINUE                                                           00001360
C                                                                        00001370
C  FIND BACKFLOW IF PRESENT ON REAR PORTION OF CYLINDER TOP AND          00001380
C  BOTTOM.                                                               00001390
C                                                                        00001400
```

```
      IF (NV.EQ.0) GO TO 106                                                    00001410
      DO 116 I=1,2                                                              00001420
      DO 117 J= 1,61                                                            00001430
      IF (I.EQ.1) Z= ZCRP(J)                                                    00001440
      IF (I.EQ.2) Z= CONJG(ZCRP(J))                                             00001450
      CALL CVELP(Z,0,0,CV)                                                      00001460
      UI= REAL(CV)                                                              00001470
      A   = CABS(CV)                                                            00001480
      IF ((UI.LE.0.0).AND.(A.GT.VELM(I+2))) GO TO 118                           00001490
      GO TO 117                                                                 00001500
  118 VELM(I+2)= A                                                              00001510
      IF (A.LE.EV) VELM(I+2)= 0.0                                               00001520
      IF (A.GT.EV) NVM(I+2)= ((-1)**(I+1))*(61-J)                               00001530
  117 CONTINUE                                                                  00001540
  116 CONTINUE                                                                  00001550
C                                                                               00001560
C  CALCULATE NASCENT VORTEX PLACEMENT ANGLE TO BE USED IN FUTURE                00001570
C  CALCULATIONS.                                                                00001580
C                                                                               00001590
  106 AVN(1)=  95.0                                                             00001600
      AVN(2)= -95.0                                                             00001610
      IF (VELM(3).GT.EV) AVN(3)=  55.0                                          00001620
      IF (VELM(4).GT.EV) AVN(4)= -55.0                                          00001630
C                                                                               00001640
C  CALCULATE VORTEX PLACEMENT DISTANCE (MP).                                    00001650
C                                                                               00001660
      DO 122 I=1,NB                                                             00001670
      IF (VELM(I).LE.EV) GO TO 122                                              00001680
      MP(I)= 0.045                                                              00001690
  122 CONTINUE                                                                  00001700
C                                                                               00001710
C  LIFT AND DRAG COEFFICIENTS                                                   00001720
C                                                                               00001730
      ZS1= (0.0,0.0)                                                            00001740
      ZS2= ZS1                                                                  00001750
      IF (NV.EQ.0) GO TO 110                                                    00001760
      DO 109 I=1,NB                                                             00001770
      K= 1+NIT*(I-1)                                                            00001780
      L= NV+NIT*(I-1)                                                           00001790
      DO 108 J= K,L                                                             00001800
      IF (ABS(GK(J)).LE.EG) GO TO 108                                           00001810
      CV= CMPLX(-AIMAG(CVV(J)),REAL(CVV(J)))                                    00001820
      ZS1= ZS1+GK(J)*CV                                                         00001830
      CV= CMPLX(-REAL(CVV(J)),AIMAG(CVV(J)))                                    00001840
      CV= CV/(CONJG(ZF(J))**2)                                                  00001850
      ZS2= ZS2+GK(J)*CMPLX(-AIMAG(CV),REAL(CV))                                 00001860
  108 CONTINUE                                                                  00001870
  109 CONTINUE                                                                  00001880
```

```
  110 CDCL(IT)= ZS1-ZS2                                                          00001890
      WRITE (6,3)                                                                00001900
      WRITE(6,1) IT,TP(IT),DTP,MPA,MZD,NDTP,(NVM(I),AVN(I),VELM(I),MP(I)         00001910
     C,I=1,NB),CDCL(IT),ZS1,ZS2                                                  00001920
      IF (NV.EQ.0) GO TO 134                                                     00001930
      K= 2*NIT                                                                   00001940
      L= 3*NIT                                                                   00001950
      WRITE (6,2) (ZF(I),CVV(I),GK(I),ZF(I+NIT),CVV(I+NIT),GK(I+NIT),           00001960
     CZF(I+K),CVV(I+K),GK(I+K),ZF(I+L),CVV(I+L),GK(I+L),I=1,NV)                  00001970
      IF (NCOAL.EQ.2) WRITE (6,5) NCAN,NCL                                       00001980
      CLOCKR= -CLOCKR+(ITIME(0)*.01)                                             00001990
      CLOCKT= (-CLOCKS+ITIME(0)*.01)/60.0                                        00002000
      WRITE (6,4) CLOCKR,CLOCKT                                                  00002010
      CLOCKR= ITIME(0)*.01                                                       00002020
C                                                                                00002030
C  PLOT THE VORTICIES' LOCATIONS.                                                00002040
C                                                                                00002050
      N= IT/10                                                                   00002060
      IF (((N.EQ.NP1).OR.(IT.EQ.NIT)).OR.(TP(IT).GE.TMAXP)) GO TO 131            00002070
      GO TO 134                                                                  00002080
  131 NPAIR= NS+3                                                                00002090
      DO 175 J= 1,NPAIR                                                          00002100
      XP(J)= REAL(ZCC(J))                                                        00002110
      YP(J)= AIMAG(ZCC(J))                                                       00002120
  175 CONTINUE                                                                   00002130
      N= 1                                                                       00002140
      CALL PLOTP (XP,YP,NPAIR,N)                                                 00002150
      NPAIR= NV                                                                  00002160
      DO 133 I= 1,NB                                                             00002170
      K= NIT*(I-1)                                                               00002180
      DO 132 J= 1,NV                                                             00002190
      XP(J)= REAL(ZF(K+J))                                                       00002200
      YP(J)= AIMAG(ZF(K+J))                                                      00002210
  132 CONTINUE                                                                   00002220
      N= 2                                                                       00002230
      IF (I.EQ.NB) N= 3                                                          00002240
      IF (I.EQ.NB) WRITE(6,3)                                                    00002250
      CALL PLOTP (XP,YP,NPAIR,N)                                                 00002260
  133 CONTINUE                                                                   00002270
      NP1= NP1+1                                                                 00002280
      WRITE (6,7) IT                                                             00002290
      IF ((IT.EQ.NIT).OR.(TP(IT).GE.TMAXP)) GO TO 1291                           00002300
C                                                                                00002310
C  ADVANCE ALL VORTICIES BY DT*(U'+IV')                                          00002320
C                                                                                00002330
  134 IF (NV.EQ.0) GO TO 161                                                     00002340
      DO 124 I=1,NB                                                              00002350
      K= 1+NIT*(I-1)                                                             00002360
```

```
      L= NV+NIT*(I-1)                                                    00002370
      DO 123 J= K,L                                                      00002380
      IF (ABS(GK(J)).GT.EG) ZF(J)= ZF(J)+DTP*CVV(J)                      00002390
  123 CONTINUE                                                           00002400
  124 CONTINUE                                                           00002410
C                                                                        00002420
CC   CREATE THE NASCENT VORTICIES.                                       00002430
C                                                                        00002440
  161 NV= IT                                                             00002450
      DO 127 I= 1,NB                                                     00002460
      K= NV+NIT*(I-1)                                                    00002470
      IF (I.GT.2) GO TO 170                                              00002480
      GK(K)= ((-1)**I)* DTP*VELM(I)*VELM(I)/2.0                          00002490
      A= AVN(I)*PI/180.0                                                 00002500
      B= MP(I)+1.0                                                       00002510
      ZF(K)= CMPLX(B*COS(A),B*SIN(A))                                    00002520
      GO TO 127                                                          00002530
  170 IF (VELM(I).LE.EV) GO TO 127                                       00002540
      GK(K)= ((-1)**(I-1))*DTP*VELM(I)*VELM(I)/2.0                       00002550
      A= AVN(I)*PI/180.0                                                 00002560
      B= MP(I)+1.0                                                       00002570
      ZF(K)= CMPLX(B*COS(A),B*SIN(A))                                    00002580
  127 CONTINUE                                                           00002590
      IF (NFOR.EQ.2) GO TO 171                                           00002600
      N= NB/2                                                            00002610
      DO 172 I= 1,N                                                      00002620
      K= NV+NIT*2*(I-1)                                                  00002630
      L= K+NIT                                                           00002640
      GK(L)= -GK(K)                                                      00002650
      ZF(L)= CONJG(ZF(K))                                                00002660
  172 CONTINUE                                                           00002670
  171 CONTINUE                                                           00002680
C                                                                        00002690
CC   INDUCE NON-SYMMETRIC VORTEX SHEDDING.                               00002700
C                                                                        00002710
      IF (NNAXI.EQ.1) GO TO 163                                          00002720
      N= TP(IT)                                                          00002730
      IF (N.NE.NTNAXI) GO TO 163                                         00002740
      GK(NV+NIT)= 2.0*GK(NV+NIT)                                         00002750
  163 TP(IT+1)= TP(IT)+DTP                                               00002760
C                                                                        00002770
CC   COALESCENCE AND CANCELLATION OF VORTICIES WITHIN CLOSE PROXIMITY    00002780
CC   TO EACH OTHER.                                                      00002790
C                                                                        00002800
      IF (NCOAL.EQ.1) GO TO 129                                          00002810
      DO 144 I= 1,NB                                                     00002820
      K= 1+NIT*(I-1)                                                     00002830
      L= NV+NIT*(I-1)                                                    00002840
```

```
      DO 143 J= K,L                                                          00002850
      IF (ABS(GK(J)).LE.EG) GO TO 143                                        00002860
      DO 142 II= I,NB                                                        00002870
      NK= 1+NIT*(II-1)                                                       00002880
      NL= NV+NIT*(II-1)                                                      00002890
      DO 141 JJ= NK,NL                                                       00002900
      IF ((J.EQ.JJ).OR.(ABS(GK(JJ)).LE.EG)) GO TO 141                        00002910
      A= REAL(ZF(J)-ZF(JJ))                                                  00002920
      B= AIMAG(ZF(J)-ZF(JJ))                                                 00002930
      IF ((A*A+B*B).LE.(MZD*MZD)) GO TO 138                                  00002940
      GO TO 141                                                              00002950
  138 G= GK(J)+GK(JJ)                                                        00002960
      IF (ABS(G).LE.EG) GO TO 139                                            00002970
      Z= ZF(JJ)-ZF(J)                                                        00002980
      A= REAL(Z)                                                             00002990
      B= AIMAG(Z)                                                            00003000
      Z= ZF(J)                                                               00003010
      GP= ABS(GK(JJ))/(ABS(GK(J))+ABS(GK(JJ)))                               00003020
      ZF(J)= CMPLX(REAL(Z)+GP*A,AIMAG(Z)+GP*B)                               00003030
      NCL= NCL+1                                                             00003040
      GO TO 140                                                              00003050
  139 ZF(J)= (0.0,0.0)                                                       00003060
      NCAN= NCAN+1                                                           00003070
  140 ZF(JJ)= (0.0,0.0)                                                      00003080
      GK(J)= G                                                               00003090
      GK(JJ)= 0.0                                                            00003100
  141 CONTINUE                                                               00003110
  142 CONTINUE                                                               00003120
  143 CONTINUE                                                               00003130
  144 CONTINUE                                                               00003140
  129 CONTINUE                                                               00003150
C*****************  END OF MAIN DO LOOP. ******************************    00003160
 1291 IF (IT.GT.NIT) IT= IT-1                                                00003170
      DO 135 I=1,IT                                                          00003180
      YP(I)= REAL(CDCL(I))                                                   00003190
      IF (ABS(YP(I)).GT.2.0) YP(I)= 0.0                                      00003200
  135 CONTINUE                                                               00003210
      WRITE (6,3)                                                            00003220
      N= IT+1                                                                00003230
      YP(N)= 2.0                                                             00003240
      TP(N)= 0.0                                                             00003250
      CALL PLOTP (TP,YP,N ,0)                                                00003260
      WRITE (6,6) (TP(I),CDCL(I),I=1,IT)                                     00003270
      RETURN                                                                 00003280
    1 FORMAT (3X,'IT = ',I4,T22,'TP = ',F7.3,T42,'DTP = ',F9.5,              00003290
     CT62,'MPA = ',F9.5,T80,'MZD = ',F9.5,//,                                00003300
     CT22,'TIME STEP MULTIPLIER = ',I2,//,                                   00003310
     CT32,'TRUE MAX.',T48,'THETA',T62,'MAX. CROSS',T77,'NASCENT VORTEX'      00003320
```

```
     C,/,T32,'VELOCITY',T47,'USED IN',T62,'& BACFLOW',T79,'PLACEMENT',      00003320
     C/,T32,'LOCATION',T45,'CALCULATIONS',T62,'VELOCITIES',T77,              00003340
     C  'DISTANCE (M)',//,                                                  00003350
     C10X,'**FRONT,TOP    **',5X,I4,10X,F7.2,8X,F8.4,9X,F9.6,/,             00003360
     C10X,'**FRONT,BOTTOM**',5X,I4,10X,F7.2,8X,F8.4,9X,F9.6,/,             00003370
     C10X,'**REAR,TOP     **',5X,I4,10X,F7.2,8X,F8.4,9X,F9.6,/,             00003380
     C10X,'**REAR,BOTTOM **',5X,I4,10X,F7.2,8X,F8.4,9X,F9.6,//,            00003390
     C3X,'CDCL = ',2F8.3,8X,'ZS1 = ',2F8.3,3X,'ZS2 = ',2F8.3,/)             00003400
    2 FORMAT (/,T62,'VORTEX SHEETS',//,T9,'1. FRONT, TOP',T42,'2. FRONT,    00003410
     C BOTTOM',T75,'3. REAR, TOP',T108,'4. REAR, BOTTOM',//,1X,             00003420
     C3(3X,'POSITION',4X,'VELOCITY',3X,'GAMMA',1X,'*'),                     00003430
     C   3X,'POSITION',4X,'VELOCITY',3X,'GAMMA',//,                        00003440
     C     (1X,  2(F6.2,1X),2(F5.2,1X),F6.3,'I',                          00003450
     C           2(F6.2,1X),2(F5.2,1X),F6.3,'I',                          00003460
     C           2(F6.2,1X),2(F5.2,1X),F6.3,'I',                          00003470
     C           2(F6.2,1X),2(F5.2,1X),F6.3))                             00003480
    3 FORMAT (1H1)                                                        00003490
    4 FORMAT (//,3X,'ELAPSED COMPUTING TIME FOR INTERATION = ',           00003500
     C                                                F8.2 ,' SECONDS.',  00003510
     C '  TOTAL ELAPSED COMPUTING TIME = ',F8.2 ,' MINUTES.')              00003520
    5 FORMAT (//,3X,'TOTAL NO. OF VORTEX PAIRS CANCELLED = ',I5,'.',       00003530
     C'TOTAL NO. OF VORTEX PAIRS COALESCENCED = ',I5,'.')                  00003540
    6 FORMAT (1H1,5X,'NORMALIZED TIME',13X,'DRAG COEF.',8X,'LIFT COEF.',   00003550
     C//,(7X,F8.4,19X,F8.3,11X,F8.3,72X))                                 00003560
    7 FORMAT (/////,10X,'VORTEX POSITION PLOT FOR IT = ',I4)              00003570
      END                                                                 00003580
```

```
      SUBROUTINE CVELP (Z,NC,NJ,ZCV)                                    00003590
C*****************************************************************     00003600
C                                                                       00003610
C     PURPOSE: TO CALCULATE THE FREE CONVECTIVE VELOCITY AT ANY POSITION,00003620
C              ON OR OUTSIDE THE CYLINDER.                              00003630
C                                                                       00003640
C     PARAMETERS:                                                       00003650
C                                                                       00003660
C   Z      = NORMALIZED POSITION ON OR OUTSIDE CYLINDER FOR WHICH THE   00003670
C            VELOCITY IS DESIRED.                                       00003680
C   NB     = NUMBER OF VORTEX BIRTH AREAS.                            00003690
C   NV     = MAXIMUM NO. OF NASCENT VORTICIES SHED IN ONE OF THE BIRTH  00003700
C            AREAS. CALCULATED AS THOUGH NO COALESCENCE OR            00003710
C            CANCELLATION WERE BEING DONE. .LE. TO NIT/NB .           00003720
C   NC     = 0,IF Z IS NOT A REAL VORTEX LOCATION.                    00003730
C            1,IF Z IS A LOCATION FOR A REAL VORTEX.                  00003740
C   NJ     = IF NC = 1, THEN THIS INDICATES THE 'J' POSITION IN THE   00003750
C            ZF (OR GK) ARRAY IN WHICH Z IS LOCATED.                  00003760
C   ZCV    = OUTPUT VELOCITY AT Z.                                    00003770
C                                                                       00003780
C*****************************************************************     00003790
      IMPLICIT  REAL (A-H,O-Y),INTEGER (I-N), COMPLEX (Z)               00003800
      COMMON /ALPHA/ZF(400),GK(400),EG,EGS,PI,NIT,NB,NV                 00003810
      A= REAL(Z)                                                        00003820
      B= AIMAG(Z)                                                       00003830
      IF ((A*A+B*B).LE.EGS) GO TO 105                                   00003840
      ZS1= (1.C/(Z*Z))-CMPLX(1.0,0.0)                                   00003850
      ZS2= (0.C,0.0)                                                    00003860
      ZS3= ZS2                                                          00003870
      IF ((NB.LE.0).OR.(NV.LE.0)) GO TO 104                             00003880
      DO 103 I= 1,NB                                                    00003890
      K= 1+NIT*(I-1)                                                    00003900
      L= NV+NIT*(I-1)                                                   00003910
      DO 102 J= K,L                                                     00003920
      IF (ABS(GK(J)).LE.EG) GO TO 102                                   00003930
      IF ((J.EQ.NJ).AND.(NC.EQ.1)) GO TO 101                            00003940
      ZSU= Z-ZF(J)                                                      00003950
      A= REAL(ZSU)                                                      00003960
      B= AIMAG(ZSU)                                                     00003970
      IF ((A*A+B*B).LE.EGS) GO TO 101                                   00003980
      ZS2= ZS2+(GK(J)/ZSU)                                              00003990
  101 ZSU= Z-(1.C/CONJG(ZF(J)))                                         00004000
      A= REAL(ZSU)                                                      00004010
      B= AIMAG(ZSU)                                                     00004020
      IF ((A*A+B*B).LE.EGS) GO TO 102                                   00004030
      ZS3= ZS3+ (GK(J)/ZSU)                                             00004040
```

```
102 CONTINUE                                                                    00004050
103 CONTINUE                                                                    00004060
104 ZS4= ZS2-ZS3                                                                00004070
    ZC= CMPLX(-AIMAG(ZS4),REAL(ZS4))/(2.0*PI)                                   00004080
    ZS4= ZS1+ZC                                                                 00004090
    ZCV= CMPLX(-REAL(ZS4),AIMAG(ZS4))                                           00004100
    RETURN                                                                      00004110
105 WRITE (6,1) Z,NB,NV    ,NJ,NC                                               00004120
    RETURN                                                                      00004130
  1 FORMAT (1H1,3X,'***** ATTEMPT TO CALCULATE A VELOCITY AT CENTER OF          00004140
   C CYLINDER. *****',//,3X,'Z = ',2E15.8,3X,'NB = ',I3,3X,'NV = ',             00004150
   CI4,3X,              'NJ = ',I4,3X,'NC = ',I3,/,1H1)                         00004160
    END                                                                         00004170
```

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13. ABSTRACT

The problem of separated flow about a circular cylinder is studied analytically through the use of two potential flow models. Following a detailed review of the present state of knowledge, mathematical formulation of the models and appropriate computer programs are presented. These models are shown to be capable of simulating the asymmetric vortex shedding.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	W
Drag on circular cylinder						
Vortex sheet						
Vortex core						
Forces on bluff bodies						
Separated flow						
Computer						
Impulsively started flow						
Shear layer development						